TNB Wars

Due September 25, 2017 by 11:59 PM

To receive full credit, you must adhere to all project guidelines on the APPM 2450 course webpage under the Projects tab, in addition to these instructions. You need to submit a clearly written report in .pdf form as well as **all** of your code files to D2L by the due date. Do not submit .zip files.

Very many years ago in a part of the universe not so close to here ...

Welcome Commanders. We are finalizing plans for a top secret operation code named TNB. Your mission is to infiltrate Xanadar IV and recover an asset with intelligence that is vital to the success of the Rebellion. Progress has been complicated by Xanadar's automated ion cannon defense system. We have provided an enemy shuttle that will enable you to land on the planet without incident. The main difficulty will be escaping the planet alive.

Enemy forces will be alerted to your presence shortly after your shuttle launches from the planet's surface. Once you reach 10 km above the surface, several ion cannons will be able to target your vessel. The cannons will be able to fire on you until you are well out of the atmosphere. Although an ordinary fighter pilot might be able to escape easily, you will be fleeing in an aging transport shuttle. Even if you manage to evade incoming fire, excessive acceleration could tear the shuttle apart before you exit the upper atmosphere. This is no mission for an ordinary pilot.

Fortunately, you have been assigned the best pilot in the rebellion. She has planned an unorthodox escape route which she believes will fool the automated defense system. We need you to analyze the escape trajectory using your knowledge on the TNB frame, space curves, and vectors to determine whether or not it is viable. Your very lives depend on this analysis! High Command will use your results to determine if the mission proceeds.

1 Introduction

You do not need to reiterate the previous information in your report, but you may summarize it briefly. Instead, your introduction should define the unit tangent vector **T**, the unit normal vector **N**, the binormal vector **B**, curvature κ, torsion τ, the tangential component of acceleration a_T, and the normal component of acceleration a_N. High command is not familiar with these concepts, so provide an explanation in words. You may use formulas, but these should supplement your description. Feel free to refer to pages 599-603 of Stewart. However, the language in your explanation should be your own. Torsion is mentioned on page 605 of Stewart, but you may wish to find another definition elsewhere.

2 The Escape Trajectory

For the rest of this project, you will be considering the following proposed escape trajectory

$$\mathbf{r}(t) = \left\langle 20\left[\cos\left(\frac{8\pi}{15}t\right)\right]^2, 10\sin\left(\frac{4\pi}{15}t\right), \frac{16}{675}t^3 \right\rangle,\ 0 \leq t \leq 15,$$

where the position is in kilometers and the time is in minutes. We will ask you to generate a 3D parametric plot of the trajectory and you may want to look at that now. The Mathematica notebook 3D Plots, Graphics, and Curves from APPM 2450 will be very helpful in making your plots and calculating various aspects of the TNB frame. Please refer to it.

Make the following preliminary calculations and figures:

(a) Plot the instantaneous speed, $|\mathbf{v}(t)|$, for $0 \leq t \leq 15$. (We recommend calculating speed as $\sqrt{\mathbf{v}(t) \cdot \mathbf{v}(t)}$ instead of using **Norm** or other functions in Mathematica. The result will be easier to use.)

(b) Plot the curvature, $\kappa(t)$, for $0 \leq t \leq 15$. (Mathematica plots by making sample points and connecting the dots. Sometimes Mathematica does not use enough points to accurately capture a graph. You might adjust the number of **PlotPoints** to make sure Mathematica plots the graph correctly.)

(c) What is the total distance along the shuttle route from $t = 0$ to $t = 15$ minutes? What is the direct distance? (Use **NIntegrate**. When defining a function using NIntegrate with a variable upper bound, use the delayed assignment operator :=. For example:

```
In[3]:=    f[t_?NumericQ ] := NIntegrate[x^2, x, 0, t];
```

defines a function that is evaluated anew every time it takes input.)

(d) Does the average speed exceed 20 km/min during the first 15 minutes?

3 Maintaining Shuttle Integrity

As stated previously, the escape shuttle is more delicate than the military spacecraft that you are familiar with. Excessive speed, curvature, or acceleration could tear the shuttle apart. You must guarantee that the escape trajectory satisfies the following safety criteria:

- The shuttle speed, $|\mathbf{v}(t)|$, should not exceed 37.5 km/min for more than 2 seconds in the first 15 minutes.
- The curvature of the trajectory should not exceed 2000 km^{-1} at any time in the first 15 minutes.
- The magnitudes of the tangential and normal components of acceleration should not exceed 120 km/min^2 at any time in the first 15 minutes.

To do this, answer the following questions in your written report:

(a) For how many seconds does the speed exceed 37.5 km/min in the first 15 minutes? (We recommend using the **FindRoot** command in Mathematica to answer this question. You will need to provide FindRoot with a guess for when $|\mathbf{v}(t)| = 37.5$. The plot of $|\mathbf{v}(t)|$ should help.)

(b) What is the maximum curvature during the first 15 minutes of flight? (We recommend using **FindMaximum** in Mathematica.)

(c) Plot $|a_T(t)|$ and the horizontal line $y = 120$ km/m^2.

(d) Plot $|a_N(t)|$ and the horizontal line $y = 120$ km/m^2.

Does the proposed the escape trajectory meet the safety criteria? Write a paragraph that justifies your answer using the plots and calculations made so far.

4 Evading the Ion Cannons

In this section, we will assume that the surface of the planet can be modeled locally as a plane. There are three ion cannons in range of the shuttle. Once the shuttle is 10 km above the surface, the ion cannons will begin to fire beams in straight line trajectories. Table 1 predicts the ion cannon locations, firing times, and constant beam velocities based on captured enemy software.

Ion Cannon Location (km)	Firing Time (min)	Beam Velocity (km/min)
$(40, 0, 0)$	$t = 7.5$	$\langle -40, 7, 15 \rangle$
$(0, -30, 0)$	$t = 8$	$\langle 20, 20, 135/32 \rangle$
$(25, 20, 0)$	$t = 9$	$\left\langle -20, -20 + 5\sqrt{3}, 640/27 \right\rangle$

Table 1: Probable ion cannon locations, firing times, and ion beam velocities.

The shuttle is not designed to take fire from certain directions. It can sustain one direct collision from an ion beam as long as the beam is not in the osculating or normal planes of the shuttle trajectory at the time of impact. Ensure that the shuttle will survive the automated ion cannon defense system by answering the following questions:

(a) Parametrize the straight line taken by the beam from each ion cannon.

(b) Do any of the beams collide with the shuttle? If yes, at what time and what position do they collide?

(c) Do any of the beams intersect the shuttle trajectory when the shuttle is not present? If yes, at what time and what position do the trajectories intersect? (We recommend using **Reduce** in Mathematica.)

(d) If an ion beam does collide with the shuttle, find the equations of both the normal plane and the osculating plane of the shuttle trajectory at the time of impact and determine whether the line of the ion beam lies in either of these planes. Give each equation in the form $ax+by+cz = d$.

Evaluate the coefficients a, b, c, and d in each equation to four decimal places.

(e) If an ion beam does collide with the shuttle, make a plot that includes the following:

   (i) The escape trajectory $\mathbf{r}(t)$ from 2.5 minutes before impact until to 2.5 minutes after impact.

   (ii) A sphere representing the shuttle at the time of impact.

   (iii) A red line representing the straight line trajectory of any ion cannon beam that collides with the shuttle.

   (iv) Circles of radius 10 km around the shuttle representing both the normal and the osculating planes at the time of impact.

   (v) Vectors of length 10 km in the direction of the tangent, normal, and binormal vectors. Color them blue, green, and red, respectively. (Again, the Mathematica notebook 3D Plots, Graphics, and Curves from APPM 2450 will be very helpful.)

5 Conclusion

Assuming our information is accurate, will the proposed escape trajectory allow the shuttle to survive the automated ion cannon defense system? Write a conclusion in a few paragraphs that justifies your answer using the plots and calculations above. Would any of this analysis change if the path of the shuttle trajectory were parametrized differently? This could occur if the shuttle traveled at a different speed along the curve. Specify which of the quantities that you calculated would be affected by a different parametrization.

Your final recommendations should address all questions above and be given in a formal report. This means that it should be in paragraph format, **not a list of answers**. Imagine that you are submitting the report to someone familiar with the proposed mission, but who hasn't seen these instructions and therefore doesn't know what "Question 3(a)" is.