Solar Panels and Optimization

Due Thu, Nov. 2 2017

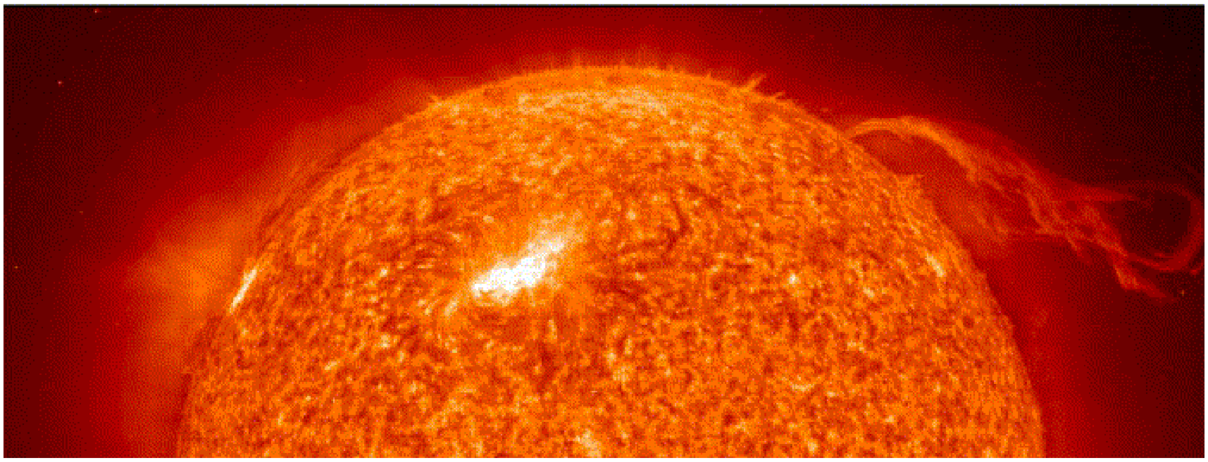

Image of the Sun. The image was taken from the astrophysics department at Stanford University.

Introduction

In this lab you will analyze how the efficiency of a solar panel depends on the season and its orientation. The solar radiation falling on a tilted plane (such as a solar panel) depends on numerous factors: the orientation of the plane, the time of the day, the season, the weather, etc. In the next few sections you will learn more about how this affects the power we can receive from solar panels.

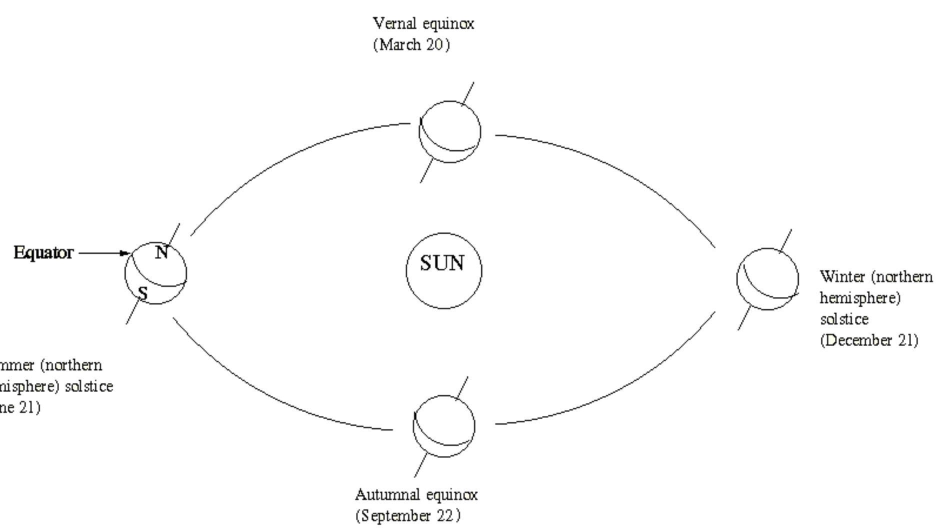

Figure 1: Sketch shows the Earth orbiting the Sun and the position of the Earth during different seasons.

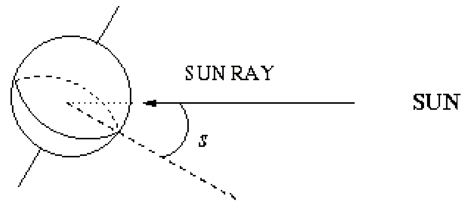

Figure 2: The angle s at the equator. The angle s is measured between the sun ray and the plane of the equator at noon. This implies that at summer solstice $s = 23°$, at winter solstice $s = -23°$ and the vernal/autumnal equinox $s = 0°$.

The Earth and the Sun

The Earth's orbit about the Sun is almost circular so for this lab we will assume that the distance to the Sun is constant. However, the axis of the Earth is tilted relative to the plane in which the Earth moves around the Sun. The angle between the axis and a normal to this plane is roughly 23°. As indicated in Figure 1, the north pole points away from the Sun during the northern hemisphere winter, and points toward the Sun during the northern hemisphere summer. Now let us imagine that we stand on the equator. We introduce the angle s as illustrated in Figure 2. In words, the angle s determines the angle between you (if you are standing straight up on the equator at noon) and a line from the center of the Earth to the center of the Sun. By "noon", we mean the time of the day when the Sun is at its highest position in the sky. As we can see from Figure 2, $s \approx 23°$ on June 20 (summer solstice) and on March 19th (vernal equinox) $s = 0°$. Hence, we can think of s as describing the time of the year. This angle s will be of great importance in the lab, so let us summarize its values in a table.

Table 1: Angle s for Times of Year

Date	s	Increasing/Decreasing
Dec. 21, 2017	$s = -23°$	
Dec. 22, 2017 - March 19, 2018	$-23° < s \leq 0°$	Increasing with time
March 20, 2018 - June 20, 2018	$0° < s \leq 23°$	Increasing with time
June 21, 2018 - Sept. 21, 2018	$23° > s \geq 0°$	Decreasing with time
Sept. 22, 2018 - Dec. 20, 2018	$0° > s \geq -23°$	Decreasing with time

IMPORTANT: The functions we use later will require that we convert degrees to radians. For example, for 23 degrees:

$$23° = \frac{23\pi}{180}\text{radians} \approx 0.4 \text{ radians}$$

Radiation incident on a tilted plane

In this lab we will be interested in the solar radiation incident on a tilted plane. Consider a solar panel that maintains a fixed angle with respect to the ground throughout the entire day. Let u be the angle between the plane and the ground. If $u = 0°$, then the plane is lying on the ground and if $u = 90°$ the plane is standing up vertically. Also, if u is negative, the solar panel is facing south, and if u is positive, it is facing north. Understanding this might give you a better idea of what's going on when you look at your 3D plot in question 2.

We will also introduce an angle t to be an angle proportional to the time of day such that $t = -90° = -\frac{\pi}{2}$ radians at dawn, $t = 0°$ at noon, and $t = 90° = \frac{\pi}{2}$ radians at dusk. If I_0 is the intesity of solar radiation (measured in $\frac{W}{m^2}$, where $W = \frac{J}{s}$ incident on the ground, then the intensity of radiation on a plane, I_p, tilted at an angle u at time of day and year corresponding to t and s respectively is given by the function:

$$I_p(s, t, u) = I_0(\cos s \cos u \cos t - \sin s \sin u)$$

Additional Factors

There are more factors that affect the radiation on the plane. There is also an absorption factor which is strongly dependent on the distance the sun's rays have traveled through the atmosphere before reaching the ground. The law describing absorption is called Beer's law. Let us describe the absorption with a function $A(s, t)$ such that $0 < A(s, t) \leq 1$. Here $A(s, t) = 1$ means that all the sunlight passes through the atmosphere (no absorption), and the absorption of light by the atmosphere increases with decreasing values of $A(s, t)$. Hence a low value ($A(s, t) \approx 0$) means that almost no sunlight makes it through (very high absorption).

We will also take different weather patterns into account since the solar radiation on the ground varies with cloudiness. Let us describe the cloudiness with a function $C(s, t)$ such that $0 < C(s, t) \leq 1$. Here $C(s, t) = 1$ means that there are no clouds, and the cloudiness increases with decreasing values of $C(s, t)$. Hence a low value ($C(s, t) \approx 0$) means that the sky is covered with thick black thunder clouds. The total amount of energy received per square meter and per day is now given by the following equation:

$$W(s, u) = \int_{t_{min}}^{t_{max}} A(s, t)C(s, t)I_p(s, u, t)dt$$

Here t_{min} and t_{max} are the angles that were discussed above. These angles limits represent the time of sunrise and sunset respectively. These times vary over the year, however, on the equator these times are relatively constant. To simplify the mathematics, we will assume that for our purposes we can use $t_{min} = -\frac{\pi}{2}$ and $t_{max} = \frac{\pi}{2}$ throughout the year. This is an approximation. For a more careful analysis of the problem, we should let these times depend on the season, i.e. on s.

Problem Statement

The Boulder company "Solar Power Inc." (invented for this lab) has asked you to do a consulting job for them. They will explore solar panels on the pacific island "Suluclac" which is located on the equator. A meteorologist has described their weather by the cloudiness function:

$$C(s,t) = \frac{3 - (1 + (s - 0.2)^2)\cos^2 t}{3}$$

where s and t are the angles related to the time of year and day as previously defined. A physicist has derived the following formula describing the energy (measured in $\frac{kWh}{m^2 day}$) where $1\ kWh = 3.6 \times 10^6 J$ collected by the solar panel each day:

$$W(s,u) = 1 + (1 + 0.65s - 1.2s^2 - 0.4s^3 + 0.35s^4)\cos u + (1.4s - 0.4s^2 - 1.5s^3 - 0.35s^4)\sin(u)$$

Note that this equation is only valid when s and u are given in radians. This energy equation has already incorporated effects due to absorption and cloudiness, so the function $W(s,u)$ is already in the form you will use for this lab. Also remember that the angle u of the solar panel is fixed on a given day. We are **not** working with a panel that moves throughout the day to follow the sun by hour. A panel like this might collect more solar energy, but it also requires a more expensive mechanical set up, so a solar panel with a fixed angle u, which may be changed manually on, say, a daily or weekly basis to at least account for the time of year, is often more practical.

The company now wants you to answer the following questions.

1. Create a labeled 3D plot of $C(s,t)$ over the domain $D = \{-0.4 \leq s \leq 0.4, -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}\}$, which will help you visualize average cloudiness. Use the information on $C(s,t)$ from the introduction (under "Additional Factors") along with your plot to describe in words how the cloudiness varies on Suluclac over the course of the average day and over the course of a year.

2. In the domain D, find the critical point(s) of $W(s,u)$ and compute the energy at the point(s).

   (a) First create a 3D plot of $W(s,u)$ with the given domain to get an idea of what is going on.

   (b) Create a contour plot of $W(s,u)$ to find where the gradient is zero in order to determine the initial guess for a possible critical point.

   (c) Use the Mathematica command `FindRoot` to find the points at which the gradient of $W(s,u)$ is the zero vector. In order to do this, you will need to give the FindRoot function an initial guess for a critical point so that it will search in that local area.

   (d) Classify the critical point(s) as local maxima, local minima or saddle points.

   (e) Compute the energy at these points.

3. Next, we will look for extreme points that may occur on the edges of our domain by fixing the value of one variable at the edge of its boundary to form a function that depends only on the variable that is not fixed.

   (a) Find all extreme points on the edges. As an example, s, the angle proportional to the time of year is bounded below by -0.4 at winter solstice and above by 0.4 at summer solstice. If we fix s to be 0.4, then we will look for the local max of the function $W(0.4, u)$ by taking its derivative and setting it to zero. Notice that when we plug in 0.4 for s in the two variable function $W(s, u)$, we end up with a single variable function $W(0.4, u)$. We can then use `FindRoot` on $\frac{d}{du}W(0.4, u)$ to find possible critical points. If we fix s to be -0.4, we look for the local max of $W(-0.4, u)$. It works the same way if we fix values of u on the other two edges. If it appears obvious to you that a particular edge will have no critical points, then you don't have to do the derivative test on that edge. Explain why you can see this from your 3D Plot.

   (b) Use your 3D plot to explain if the points you found in part (a) are maxima, minima, saddle points, etc.

4. Compute the energy at the points that you found above, as well as at the four corners of the boundary.

   (a) Plug in the values you just found in Question 3 to the $W(s, u)$ function. Then plug in corner values. For example, check the corner value of $W(-0.4, \frac{\pi}{2})$.

   (b) Specify if these values (points from Question 3, and corner points) are global maxima or global minima in the domain of the function by comparing them to the energy at the critical point(s) found in Question 2.

   For example, if the value at one of the corners turns out to be smaller than any of the other values observed, then the value at that corner would be the global minimum for the *given* domain. Also important to note is that if we were to increase the size of the domain, we might have different global extrema, but for this problem we are only focused on the domain given at the beginning of the lab because it is most relevant to our application. **Make sure you've defined your domain correctly!**

5. Recall that the angle u refers to the angle that a fixed solar panel makes with the ground. The solar panel doesn't move on its own. It will stay at the same angle until someone manually changes it. Suppose we hire someone to change the angle u of the solar panel on a regular basis, maybe once at the end of each day, so that on any given day we end up collecting as much energy as we can based on the angle s that the equator makes with the incoming sun rays.

   During what season of the year will we end up collecting the most solar energy? Explain your answer, i.e. don't just say "When there is more light!".

Also, remember that the angle s is proportional to the season of the year. In radians, s goes from 0.4 at summer solstice (the longest day of the year) all the way down to 0 at autumnal equinox, then down to -0.4 at the winter solstice (the shortest day of the year), back up to 0 at the vernal equinox, and back to 0.4 at the summer solstice.

6. Look back at the 3D plot that you created in Question 2. It should be clear that for any fixed value of s, there is an optimal angle u that will maximize the amount of solar energy collected. For fixed s, finding the optimal value of u is equivalent to finding the maximum of a single variable function. In the following steps, you will find the optimal angle u for discrete values of s from -0.4 to 0.4. The Mathematica code will be a little tricky, so we will walk you thought it.

   (a) First create a table with values of s starting with $s = -0.4$ and increasing in steps of 0.02 up to $s = 0.4$. We will use the command `Table` and store the table as `svals`. The Mathematic code for this step is:

   ```
   svals = Table[s,{s,-0.4,0.4,0.02}];
   ```

   (b) Next, we want to find the values of u that will maximize the energy collected at each of the s values in our previous table. Again, we will use the `Table` command, but what we want to store is the values of u that will make $\frac{d}{du}W(s_i, u) = 0$, where s_i is one of the discrete s values in the table from (a). We will call this table `uvals`. To find the maximizing u values, we will need to use `FindRoot`. In general, you will need to take some care in selecting your initial guess at each step, but in this case an initial guess of zero will work for each root we need to find.

   ```
   uvals = Table[FindRoot[D[W[s,u],u],{u,0}],{s,-0.4,0.4,0.02}];
   ```

   You will have to have your function $W(s, u)$ defined in Mathematica as `W[s,u]` first for this code to work!

   (c) We will want to plot an optimal path in our domain and overlay this on our contour plot for $W(s, u)$. To plot our path, we will use `ListPlot` which will plot discrete points when coordinates are specified in the su-plane. We will create this overlaid plot in three steps.

      i. Create a table called `coords` containing the su-coordinates corresponding to the optimal path:

      ```
      coords = Table[{svals[[i]],u}/.uvals[[i]],{i,Length[svals]}];
      ```

      ii. Next create a plot of the optimal path. Since we have discrete points, we will use `ListPlot`:

      ```
      IdealPointsPlot = ListPlot[coords,PlotStyle->Red];
      ```

      iii. You should have already created a contour plot of $W(s, u)$ in Question 2. If you haven't already done so, store this plot. For example, go back to the code where you created the plot and type `WContours = ContourPlot[...]`. Now we will overlay our plot of the optimal path onto the contour plot:

```
Show[WContours,IdealPointsPlot]
```

(d) Does your optimal path make sense based on what you wee from your 3D graph of $W(s,u)$? Explain.

7. Suppose that the solar panels need to be replaced once per year, and suppose it takes a significant amount of time to replace them, i.e. days or weeks. Also, suppose that during the replacement process, we can't collect any solar energy. What time of year should we replace the panels? Explain.

Solar Power Inc. wants you to describe all your results in a formal written report, containing an introduction, relevant equations, answers to all questions at appropriate places within paragraphs and a conclusion.