Vertical Transport by Convection Plumes: Modification by Rotation

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Abstract. Convection, generated by destabilising buoyancy forcing, is a significant source of vertical mixing in the ocean and atmosphere. Ensembles of convective elements or plumes transport buoyancy and tracers across the convective layer. Here we examine numerical simulations of turbulent convection to identify the modifications to the vertical transports induced by strong rotation (typical of deep ocean convection). We extract the typical convection plumes from the numerical simulations using a conditional sampling compositing technique. The plume budgets of mass, heat and momentum are then compared with an entraining plume model. In the presence of strong rotation the vertical transports are significantly reduced by vigorous lateral mixing, which dilutes plume anomalies. This mixing is generated by the energetic cyclonic vortices associated with the individual plumes and the interaction between the many vortices in the plume ensemble.

1 Introduction

Deep ocean convection may be significantly affected by rotation, characterized by a low convective Rossby number $R_o = \sqrt{Ra/(TaPr)}$ where $Ra$ is the Rayleigh number, $Ta$ is the Taylor number and $Pr$ is the Prandtl number, defined as

$$Ra = \frac{g\alpha \Delta T L^3}{\nu \kappa}; \; Ta = \frac{f^2 L^4}{\nu^2}; \; Pr = \frac{\nu}{\kappa}$$

where $g$ is the gravitational acceleration, $\alpha$ is the coefficient of thermal expansion, $\Delta T$ is the temperature difference across the convecting layer, $L$ is the depth of the convecting layer, $\nu$ is the viscosity, $\kappa$ is the diffusivity and $f$ is the Coriolis parameter. In order to examine the role of strong rotation in modifying convective transports, we have carried out a series of numerical simulations of Rayleigh-Benard convection at constant $R_o = 0.75$, and comparison calculations at $R_o = 0.3$ and $R_o = \infty$. For $R_o = 0.75$ we find that for progressively higher $Ra$ the horizontally averaged temperature field tends to an asymptotic state characterized by a non-zero negative vertical gradient in the interior (Julien et al., 1996). In contrast, non-rotating Rayleigh-Benard convection shows a well-mixed interior with zero temperature gradient in the interior. The non-zero temperature gradient in the strongly rotating case suggests reduced efficiency of vertical mixing, and a suppression of buoyancy transports across the layer. Here we investigate the possibility for reduced transports further, by focusing on the plume scale budgets of temperature, momentum and mass. We extract the typical plume structure at each vertical level by compositing all plumes at this level, azimuthally averaged about the plume center (defined as the location of the vertical velocity maximum). For further details of the plume extraction procedure see Julien et al. (1999). We then integrate temperature, and momentum over the spheroidal volume defined by the half-width of the vertical velocity. The evolution of these volume-integrated quantities as the plumes travel through the convective layer are described below.

2 An entraining-detraining plume model

We examine the plume transports in terms of an entraining parcel model, adapted from Turner (1903). In such a model a turbulent parcel engulfs ambient fluid through entrainment, changing the properties of the parcel and enlarging the parcel (Morton et al., 1956). A detraining parcel loses fluid to the surroundings, in which case the parcel properties do not change, but its volume diminishes, and the environment is modified by the mixing in of plume fluid. The parcel properties may also be modified without any change in volume if subparcels...
are exchanged between the plume and exterior (Priestley, 1953). We include the possibility for all these types of parcel modification in our model.

The parcel model is defined by the following equations:

\[
\frac{dV}{dt} = \int_S v_c dS' \; \quad (2)
\]

\[
\frac{dVW}{dt} = -V \frac{\partial P}{\partial z} + \int_S w v_c dS' - \int_S w U dS' + V g T + V v \nabla^2 W \; \quad (3)
\]

\[
\frac{dVT}{dt} = \int_S T v_c dS' - \int_S T U dS' + V k \nabla^2 T \; \quad (4)
\]

Where \( V \) is the parcel volume, \( S \) is the bounding surface of the volume, \( P \) is the pressure, normalized by dividing through by \( \rho \). \( T \) is the temperature, \( v \) is the vertical velocity, and \( g' = g\alpha(T - T_0) \) is the buoyancy anomaly, where \( T_0 \) is the horizontally averaged temperature. \( \alpha \) represents the average over the parcel, \( \bar{X} = 1/V \int_X X dV' \). We have subtracted the mean temperature field from both buoyancy and pressure terms so that \( \partial P'/\partial z = \partial P'/\partial z - g' T_0 \). \( v_c \) is the entrainment velocity, and the term \( \int_S v_c Q dS' \) represents the change of a quantity \( Q \) associated with a change in volume. This could be entrainment \( v_c > 0 \) or detrainment \( v_c < 0 \). Another term \( -\int_S Q U dS' \), which we will hereafter refer to as the exchange term, represents the change of the quantity \( Q \) due to exchange between the plume and environment, without any changes in volume (as in Priestley, 1953). \( v_c \) is therefore associated with the velocity of the plume bounding surface, while \( U \) is associated with velocity across that surface. Note that this parcel model differs from the classical entraining parcel model through its inclusion of the perturbation pressure term, the momentum entrainment term and the terms representing mixing between plume and environment. By evaluating the left hand side and buoyancy, pressure-gradient and frictional terms directly, we will deduce the entrainment and exchange terms.

3 Evaluation of parcel budgets

In figure (1) we show the evaluation of the terms in the momentum and heat budget equations as a function of the height of the plume, for a typical \( Ra_e = 0.75 \) calculation. We have here calculated terms only for hot plumes emitted from the lower boundary layer. The rate of change of heat content has been calculated by fitting a 4th order polynomial to the plume heat flux as a function of plume height and then taking the first derivative of that polynomial. For all rotating plume solutions we find that the rate of change of heat content is negative over the upper part of the convective layer, implying that heat is being lost to the surroundings. This heat loss cannot be accounted for by diffusion, which is small, and therefore must be due to entrainment and mixing in of ambient cooler fluid. The residual of the heat equation is therefore associated with the heat entrainment and mixing terms.

In the momentum equation we see that the positive buoyancy anomaly tends to accelerate the plume upwards; however this acceleration is partially compensated by a decelerating pressure gradient across the parcel. The net acceleration is large and positive near the lower boundary and gradually reduces near the opposite boundary. This implies a large residual in the momentum budget, which we associate with entrainment and mixing: upward momentum is entrained near the lower boundary, but in the interior entrainment and mixing tend to decelerate the plume.

From the mass budget or change in volume of the plume, we can calculate the rate of entrainment. The entrainment velocity is often parameterized by \( v_e = \alpha \bar{W} \), where \( \alpha \), the constant of proportionality is termed the entrainment coefficient (Morton et al, 1956). In figure (2) we show the entrainment coefficient evaluated for 4 different \( Ra_e = 0.75 \) solutions, at different \( Ra \) and with different momentum boundary conditions (no slip / stress-free). We also show, for comparison, the entrainment coefficient for one non-rotating calculation and one \( Ra = 0.3 \) calculation. For all the strongly rotating solutions the entrainment coefficient decreases rapidly to zero in the interior, while for the non-rotating case \( \alpha \) remains at between 0.1 and 0.2 as found from laboratory experiments of isolated plumes (Emanuel, 1994).

We can similarly parameterize the mixing between plume and environment by a mixing coefficient \( \alpha_x \), defined by \( u_x = \alpha_x \bar{W} \) where \( u_x \) is a mixing velocity scale. If we use the entrainment coefficient calculated above to estimate the rate of change in heat content due to entrainment:

\[
\int_S T v_c dS' \approx \alpha \bar{W} S_{TE} = T_E \frac{d}{dz} \bar{W} \; \quad (5)
\]

where \( S_{TE} \) is the entrained fluid temperature.

We approximate the exchange term similarly:

\[
\int_S T U dS' \approx T_x u_x S \; \quad (6)
\]

where \( u_x \) is an exchange velocity scale, and \( T_x \) an exchange temperature scale. Since the surface averaged velocity across the thermal boundary is zero from continuity, \( u_x \) represents an r.m.s. anomaly velocity across the boundary. The the residual in the heat equation (4) can be expressed as

\[
\text{Residual} = \int_S T v_c dS' - \int_S T U dS' = T_E \frac{d}{dz} \bar{W} + \alpha_x T_x \bar{W} S \; \quad (7)
\]
To solve for $\alpha_x$ we make suitable choices for $T_E$ and $T_0$ (the temperature difference between exchanged parcels of fluid): $T_E = T_0$, the horizontally averaged temperature, and $T_2 = \bar{T} - T_0$. Then

$$\alpha_x = \frac{1}{S} \left( \text{Residual} - T_0 \frac{d}{dz}(\bar{V}\bar{W}) \right) \left( (T - T_0)(\bar{W}) \right)$$

Since the plume is warmer than its surroundings, we expect motion directed away from the plume centre to be associated with warm temperature anomalies, and motion toward the plume centre to be associated with cooler temperatures, a down-gradient lateral flux associated with negative values of both the exchange term and $\alpha_x$.

In figure (2)b we see the $\alpha_x$ estimated in this way for the $4 \cdot Ro_C = 0.75$ calculations, and one $Ro_C = 0.3$ and one $Ro_C = \infty$ calculation as before. For all the rotating runs, $\alpha_x$ is negative and of greater magnitude than the entrainment coefficient. Hence for the rotating runs, warm fluid from the plume is exchanged with cooler fluid from the exterior, resulting in a down-gradient flux which decreases the plume heat content.

Without rotation, there is a positive residual in the heat equation over most of the domain. If we assume that entrained fluid is at the mean ambient temperature, which is close to zero, this implies a positive exchange coefficient $\alpha_x$, as shown in figure (2)b, implying a counter-gradient heat flux. An alternative explanation is that entrainment into the non-rotating plumes is preferentially of hotter fluid, such that in smaller hot plumes engulfed by the larger hot plumes.

### 4 Causes of enhanced mixing in rotating plumes

Our plume budget analysis shows that compared to non-rotating plumes, rotating plumes have reduced entrainment over the interior of the convective layer. However the plume buoyancy continues to be diluted in the absence of entrainment, through mixing and exchange of subparcels with the environment. The causes of entrainment have been well studied in the context of isolated non-rotating plumes - 3-dimensional eddies within the plumes engulf filaments of fluid into the plume, where they are then mixed with the plume fluid (Papantoniou and List, 1986). Other authors have suggested that the suppression of entrainment with strong rotation is a consequence of angular momentum constraints which prevent the horizontal expansion of the plumes from continuing beyond a limiting radius (Fernando and Ching, 1993; Helfrich, 1994). In rotating plumes there are strong vertically elongated cyclonic vortices generated by the convergence of fluid into a plume (Jones and Marshall, 1993; Maxworthy and Narimousa, 1994). Hence despite the suppression of the entrainment, the motion is not 2-dimensional: vertical horizontal motion occurs in conjunction with the vertical motion driven by buoyancy. A field of cyclonic vortices in a 2-dimensional flow will efficiently mix the fluid outside of the vortices; if vortices merge, filaments of vortex fluid will also be mixed into the environment. Similar interactions take place between the vortices associated with plumes, efficiently stirring the fluid in the horizontal while the plume transports heat across the layer. Hence the continued dilution of buoyancy of rotating plumes by mixing with the environment is a consequence of the vortices associated with the plumes.

### 5 Conclusions

We have shown, by examining the budgets of mass, heat and momentum for composite plumes extracted from solutions of rotating Rayleigh-Benard convection that entrainment of fluid into the plume is suppressed for strong...
rotation and replaced by mixing of properties with the environment without any further change in volume of the parcel. This continued mixing between the plume and environment is most likely the result of interactions between vortices associated with plumes which stir the fluid in the horizontal plane. The mixing tends to dilute the plume buoyancy anomaly, leading to a decrease in the total heat transported by a plume as it migrates across the layer. A net effect of the decrease in buoyancy transport may be the finite negative temperature gradient which persists even at large $Ra$.

The net plume acceleration across the layer is also suppressed compared to the non-rotating case, as a result of mixing of momentum with the environment and the pressure gradient deceleration. The pressure gradient deceleration remains a considerable obstacle to closure in parameterizations of plume transports.

The modification of plume transports by rotation is likely to be important in regions of open ocean deep convection (Marshall and Schott, 1999). Plume transports have not yet been evaluated from observations, but recent measurements in the Labrador Sea (Labsea Group, 1998) which include data from parcel-following 3D lagrangian floats make this feasible. This study suggests that parameterizations of the convective fluxes in use in coarse resolution ocean models (e.g. Large et al, 1994) may need to be extended to include the effects of rotation.

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**References**


