THE GRADUATE SCHOOL
of
**THE UNIVERSITY OF COLORADO
AT BOULDER**

DISSERTATION DEFENSE
of

Henry P Romero

FOR THE DEGREE

DOCTOR OF PHILOSOPHY

Date/Time: November 17th, 2014 from 3:00 pm

Bldg./Rm: Koelbel, S130

Examining Committee Members:
Prof. Mahesh Varanasi, Prof. Juan Restrepo, Prof. Vanja Dukic, Prof. Jem Corcoran, Prof. Clifford Mullis

OUTLINE OF STUDIES

Major Field: Applied Mathematics

BIOGRAPHICAL NOTES

Henry Romero received his BS in Applied Mathematics and Electrical Engineering from the University of Colorado at Boulder in May of 2007. From 2007-2009, he worked at the National Institute of Standards and Technology, while studying for a Master's degree in Applied Mathematics. Since 2010, he has been working under the supervision of Prof. Mahesh Varanasi on fundamental problems within Network Information Theory which are motivated primarily by applications in wireless communication. His career interests lie in using principled mathematical reasoning to provide insight into complex engineering and scientific questions.

THESIS

Complete title of thesis:
Fundamental Limits of Network Communication with General Message Sets: A Combinatorial Approach

Faculty Advisor Prof. Mahesh Varanasi

ABSTRACT

The classical theoretical framework for communication networks is based on the simplifying assumption that each message to be sent is known only to one transmitter and intended only for one receiver. Modern communication protocols reflect this framework by treating the physical layer as a network of individual links. However, this wireline view of wireless communications fails to account for the heterogeneous nature of network demands, consisting of both unicast and multicast services, and can fail to leverage the inherent broadcast advantage of the wireless medium.

This thesis extends the classical framework of a private-message interface to the physical layer to one with both private and common messages. A key difficulty, in both the description and analysis of a communication model with general messages sets, is that there are combinatorially many message possibilities. With order-theoretic language and tools from combinatorial optimization and graphical models, we develop a general framework for characterizing the fundamental limits of information transfer over large many-to-one (multiple access) and one-to-many (broadcast) communication channels with general message sets. In particular, achievable regions are proposed for arbitrary such channels. For the multiple-access channel, the achievable region is optimal, and the order-theoretic perspective both unifies and extends previous results. For the broadcast channel, the region is specialized to an inner bound to the Degree of Freedom region, a setting where it is provably optimal in select cases.

This thesis provides fresh insights into the long-standing random coding technique of superposition coding to arrive at these results. Governing constraints on reliable communication through superposition coding are shown to be polymatroidal over a lattice of subsets that may not be the boolean lattice of *all* subsets. Permissible input distributions for superposition coding are concisely characterized through directed graphical models of conditional dependencies. The two-user interference channel is also addressed, where the state-of-the art is extended from the case with two private messages to one with an additional common message.