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Presenter

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Title

Toward a general solution of the three-wave resonant interaction equations

The resonant interaction of three wavetrains is one of the simplest forms of nonlinear interaction for dispersive waves of small amplitude. This behavior arises frequently in applications ranging from nonlinear optics to internal waves through the study of the weakly nonlinear limit of a dispersive system. The slowly varying amplitudes of the three waves satisfy a set of integrable nonlinear partial differential equations known as the three-wave equations. So far, these universally occurring equations have been solved in only a limited number of configurations. For example, Zakharov and Manakov (1973, 1976) and Kaup (1976) used inverse scattering to solve the three-wave equations in one spatial dimension on the real line. Similarly, solutions in two or three spatial dimensions on the whole space were worked out by Zakharov (1976), Kaup (1980), and others. The known methods of analytic solution fail in the case of periodic boundary conditions, although numerical simulations of the problem typically impose these conditions.

We hope to find a general solution to the three-wave equations, which has the advantage of being compatible with a wide variety of boundary conditions and any number of spatial dimensions. To find the general solution of an nth order system of ordinary differential equations, it is sufficient to find a function that satisfies the ODEs and has n constants of integration. The general solution of a PDE, however, is not well defined and is usually difficult, if not impossible, to attain. In fact, there is only a small number of PDEs with known general solutions. We present a method to construct the general solution of the three-wave equations using a Painlevé-type analysis. For now, we consider a convergent Laurent series solution (in time), which contains five real-valued functions (in space) that are arbitrary except for some differentiability constraints. A full general solution of the problem would involve six such functions.