APPM 4360 Homework 3 (Due Feb 10)

1. Derive the formulas

   (a) $\arcsin z = -i \log(iz + (1 - z^2)^{1/2})$

   (b) $\frac{d}{dz} \arcsin z = \frac{1}{(1-z^2)^{1/2}}$

2. Find the radius of convergence of the following series. Hint: For n large, $n! \approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$.

   (a) $\sum_{k=1}^{\infty} \frac{k!}{k^k} z^k$

   (b) The Taylor series of $\frac{1+z}{2-z}$ around $z_0 = i$

3. Find the complete Taylor series expansions around $z = 0$ of the following functions, and tell in which regions they are valid.

   (a) $\frac{1}{2-z^2}$

   (b) $\frac{e^{z^2}-1-z^2}{z^3}$

4. The Bernoulli numbers $\{B_k\}_{k=0}^{\infty}$ are defined such that $\frac{z}{e^z-1} = \sum_{k=0}^{\infty} \frac{B_k}{k!} z^k$.

   (a) Find the first three Bernoulli numbers.

   (b) Find the radius of convergence of the series.

5. Find all singularities of the following functions. Identify their types (cf. Section 2.4 of the course text book).

   (a) $f(z) = \frac{\cos z - 1}{z^4}$

   (b) $f(z) = z^{3/2} e^{1/(z-1)}$

   (c) $f(z) = 1/\sin \frac{1}{z}$

6. Find the location of the branch points and discuss possible branch cuts for the following functions. Don't forget to check if $z = \infty$ is a branch point, or not.

   (a) $f(z) = (z^2 + 1)^{1/2}$

   (b) $f(z) = (z^2 + 1)^{1/3}$