

APPM 4360 Homework 2 (Due Feb 3)

1. Are the following functions analytic? Check using the Cauchy-Riemann (C-R) equations? If they are, write them down as analytic functions of z .

(a) $f(x, y) = y^3 - 3x^2y + i(x^3 - 3xy^2 + 2)$

(c) $f(x, y) = e^{x-iy}$

(b) $f(x, y) = \frac{z}{z^4+1}$

(d) $f(x, y) = e^{1/(z-1)}$

2. Let $f(x, y) = u(x, y) + iv(x, y)$ be an analytic function, where $u(x, y)$ is given below. Find $v(x, y)$, the harmonic conjugate of $u(x, y)$, and express the function in terms of z .

(a) $u(x, y) = \frac{y}{x^2+y^2}$

(b) $u(x, y) = \cos x \cosh y$

3. Express the Cauchy-Riemann equations in terms of polar coordinates.

4. Let $g(z)$ be some analytic function. Determine if $f(z) = \overline{g(\bar{z})}$ is analytic.

5. Use either the Matlab or the Mathematica code given in the course text book (and on the course web page), or use any other programming language to plot the real part, imaginary part, magnitude (modulus) and argument (phase portrait) of the following functions:

(a) $f(z) = e^z$

(d) $f(z) = 1/z$

(b) $f(z) = \log z$

(e) $f(z) = \arctan z$

(c) $f(z) = z^6$

(f) $f(z) = \tan z$