APPM 4360 Homework 1 (Due Jan 27)

1. Find the polar form of the following complex numbers

   (a) $\frac{1}{(1-i)^8}$

   (b) $\frac{1+i}{\sqrt{1+i\sqrt{3}}}$

   (c) $\left(\frac{1+i}{1-i}\right)^4$

   (d) The roots of $z^7 - 128 = 0$

   (e) $\frac{1+i\sqrt{3}}{1+i}$. Use your result to compute $\cos\frac{\pi}{12}$.

2. Express each of the following numbers in the form $a + bi$, where a and b are real

   (a) The roots of $(z + \frac{i}{2})^4 = 16$

   (b) The roots of $z^2 + iz - i = 0$

   (c) $\left(\frac{1-i}{1+i}\right)^4$

   (d) $i^{1/2}$

3. Draw the set of points that satisfy

   (a) $\text{Im}\,(z + 2) = 3$

   (b) $|z - i| < 2$

   (c) $|z - i + 2| = |z + 2i - 1|$

   (d) $|z - 1| + |z + 1| = 3$

4. Let z and w be any two complex numbers. Prove the relations

   (a) $z - \bar{z} = 2\ i\ \text{Im}\,(z)$

   (b) $\text{Re}\,(z) \leq |z|$, where $\text{Re}\,(z)$ is the real part of z

   (c) $|w\bar{z} + \bar{w}z| \leq 2\ |z\ w|$

5. Show that, for the stereographic projection, a circle in the z-plane corresponds to a circle on the sphere. Hint: a circle on the spheres is given by the intersection of the sphere with a plane

$$AX + BY + CZ - D = 0$$