APPM 3570: Homework Set 9

Due Wed. Mar. 21, 2018

Note: To help out the LAs, please draw a grading table at the top of the first page of your homework. The table should have five rows and two columns, just like the ones drawn on your graded homework.

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1. Chapter 5 in Ross: Problems 32, 34, 36, 38, 41; Theoretical Exercises 28, 31

2. **Exponential distribution.** Half the cars sold by a dealer are lemons, and half are good. Good cars fail according to an exponential distribution with a rate of $\lambda_1 = 1/(10$ years), and lemons fail according to an exponential distribution with a rate of $\lambda_2 = 1/(2$ years).

   (a) You buy a car, not knowing whether you received a good car or a lemon. What is the p.d.f. for T, the time it takes the car to fail?

   (b) Assume your car still works after five years, and use Bayes' Theorem to compute the probability you bought a good car.

   (c) If your car fails after two years, what is the probability you bought a lemon?

3. **Gamma distribution.** You order a pizza from Cosmo's and the number of minutes X it takes to arrive is a Gamma distributed random variable with mean 32 minutes and standard deviation 16 minutes.

   (a) Using the results of Example 5.6a in Ross (p. 205), determine the parameters (α, λ) and full p.d.f. $f(x)$ of the corresponding Gamma distribution.

   (b) If you placed your order at 8:00pm, what is the probability you get your pizza before 8:30pm?

   (c) Your friend pulls a prank on you by having a Limburger cheese pizza delivered to your house that arrives at 9:00pm. What is the probability they ordered the pizza before 8:30pm? Assume the distribution of delivery times is the same as the result in part (a).

   (d) Compute $P(X < 2)$ using the p.d.f. derived in part (a). Explain why your answer makes sense.

4. **Rectified random variables.** For many applications in engineering and science, it is not reasonable to consider negative values of random variables. One solution to this problem is to rectify random variables by defining $[y]_+$ for any real number:

$$[y]_+ = \begin{cases} y, & y \geq 0 \\ 0, & y < 0. \end{cases}$$

   (a) Consider $X \sim U(-1, 1)$ uniformly distributed between $[-1, 1]$. Now take $Y = [X]_+$. First, compute $P(Y = 0) = P(X \leq 0)$. Use this result to compute the c.d.f. $F(y)$ of Y, and plot it.

   (b) What is $E[Y]$? Is it greater or less than $E[X]$? Explain why.

   (c) Assume Z is a standard normal random variable. Take $R = Z_+$ and compute $E[R]$.