9. Assignment 9

Due Wednesday, April 18

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(1) Implement the Crank-Nicolson scheme for the heat equation

$$\frac{\partial \phi}{\partial t} = \partial_x (a(x) \, \partial_x \phi) + f(x, t),$$

with the initial condition

$$\phi|_{t=0} = \phi_0$$

and the boundary condition $\phi(t,0) = 0$ and $\phi(t,1) = 0$. Verify the scheme on several examples. Implement an explicit scheme for same equation and verify that it is stable only if $h_t = O(h_x^2)$

(2) Implement the explicit second-order central difference scheme for the wave equation

$$\frac{\partial^2 \phi}{\partial t^2} = \partial_x (a(x) \,\partial_x \phi) + f(x,t),$$

 $\phi|_{t=0} = \phi_0,$

with the initial condition

and

$$\frac{\partial \phi}{\partial t}|_{t=0} = \phi_1,$$

where all functions are periodic in x with the period 1. Verify the scheme on several examples using an appropriate choice (explain) of step sizes h_t and h_x . Try to run the scheme with the step sizes violating the stability criterion. Describe the numerical effect.

In both problems it is suggested to use graphical display of solutions (at various moments in time). As a way of understanding ill-posed problems, reverse the direction of time in the heat equation and the Crank-Nicolson scheme and describe the effects of such reversal. Also, by changing the sign of the right hand side of the wave equation, try to solve an ill-posed initial value problem for the resulting elliptic equation. Describe the numerical effects.