APPM 3570/STAT 3100 Spring 2019 Homework 9 - Due Mar. 20

- Chapter 5, Problems 31, 40, 42.
- Chapter 5, Theoretical Exercises 19, 26.
- Let X be a random variable with CDF $F(x)$. Suppose that we want to generate random numbers with the distribution of X, but have only access to a random variable U that is uniformly distributed in $[0,1]$ (like in computer random number generators). Show that the random variable $Y = F^{-1}(U)$ has CDF $F(y)$, and therefore has the same distribution as X. This is a way to numerically generate random numbers with arbitrary distributions.
- **Extra credit - harder problem (2 points). Reconciling the Poisson and Gaussian approximations to the Binomial.**

  We have seen that a Binomial distribution can be approximated by a Poisson when $n \to \infty$, $p \to 0$, and by a Gaussian when $n \to \infty$. Therefore somehow they should agree when $n \to \infty$. In this problem we will see how this is the case when $np \to \infty$, even though at first glance the Poisson and Gaussian distributions look very different. Consider a Binomial distribution with parameters (n,p), and let $\lambda = np$.

  - (0 points. Just a tip...) Show that when $p \to 0$, the Binomial has mean λ and variance λ.
  - (1 point) Now let $k = \lambda + z\sqrt{\lambda}$. Here, z represents how many standard deviations ($\sqrt{z}$) is k away from the mean (λ). Let X be the Gaussian approximation to the Binomial. Show that as $\lambda \to \infty$,

    $$P(X=k) \to \frac{1}{\sqrt{2\pi\lambda}} e^{-z^2/2}. \tag{1}$$

  - (1 point) Now let Y be the Poisson approximation. The Poisson approximation says $P(Y=k) = \frac{\lambda^k e^{-\lambda}}{k!}$, so

    $$P(Y=k) = \frac{\lambda^{(\lambda+z\sqrt{\lambda})} e^{-\lambda}}{(\lambda+z\sqrt{\lambda})!}. \tag{2}$$

    Show that the two approximations give the same result as $\lambda \to \infty$. To show the expressions (1) and (2) have the same limit as $\lambda \to \infty$, you can use Stirling's approximation,

    $$\ln(n!) \sim n \ln n - n + \ln\sqrt{2\pi n},$$

    valid for large n (not covered in class), and keep only the leading terms in λ.

Note, as with all homework sets in this class, that you may discuss the homework problems with your classmates. However, the work you turn in must be your own – you should write your solutions on your own. Identical solutions will be considered as a violation of the Student Honor Code. Furthermore, **no work equals no credit.** Your homework should be neatly written or typed and stapled.

On the front of your homework clearly print your:

- First Name and Last Name
- Lecture number (either Section 001 or Section 002) and homework number.
- Draw a **blank grading table** with room for 3 problems, format points and a total:
- Points will be deducted if these instructions are not followed.

Remember that writing style, clarity, and completeness of explanations are always important. Justify your answers.