8. Assignment 8

Due Wednesday, April 11

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(1) A popular explicit Runge-Kutta method is defined by the following formulas:

$$\begin{aligned} k_1 &= hf(x_n, y_n) \\ k_2 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_1) \\ k_3 &= hf(x_n + \tfrac{1}{2}h, y_n + \tfrac{1}{2}k_2) \\ k_4 &= hf(x_n + h, y_n + k_3) \\ y_{n+1} &= y_n + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

Estimate the region of absolute stability of this method (calculate all intersections of the region with the real and imaginary axes).

(2) One seeks the solution of the eigenvalue problem

$$\frac{d}{dx}\left[\left(\frac{1}{1+x}\right)\frac{dy}{dx}\right] + \lambda y = 0,$$

with boundary conditions $y(0) = y(1) = 0$ by integrating, for a few values of λ, an equivalent system of two first order differential equations with initial values $y(0) = 0$ and $y'(0) = 1$, using trapezoidal method combined with Richardson's extrapolation developed in a previous assignment. Taking λ in the range $[6.7, 6.8]$, compute the value of λ for which $y(1) = 0$.

(3) Problem #54, page 459 in Atkinson's book (a simple problem on changing boundary conditions)
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