APPM 4/5560

Problem Set Seven (Due Wednesday, March 13th)

1. Cars pass a certain street location according to a Poisson process with rate λ. Bart, wanting to cross the street at that location, waits until he can see that no cars will come by in the next T time units.

   (a) Find the probability that his waiting time is zero.

   (b) Find his expected waiting time.

2. Suppose $\{N(t)\}_{t\geq 0}$ is a Poisson process with rate λ. Let T_n denote the time of the nth arrival. Find

   (a) $\mathsf{E}[T_{10}]$

   (b) $\mathsf{E}[T_{10}|T_3 = 5.4]$

   (c) $\mathsf{E}[T_{10}|N(4) = 2]$

   (d) $\mathsf{E}[T(3)|N(1) = 0]$

3. (Durrett 2.2) The lifetime of a radio is exponentially distributed with *mean* 5 years. If Ted buys a 7 year-old radio, what is the probability it will be working 3 years later?

4. (Most of Durrett 2.10) Consider a bank with two tellers. Three people, Alice, Betty, and Carol enter the bank at almost the same time and in that order. Alice and betty go directly into service while Carol waits for the first available teller. Suppose that the service times (in minutes) for each cumstomer are exponentially distributed with rate 1/4.

   (a) What is the expected total amount of time for Carol to complete her business transaction? (Include waiting and service time.)

   (b) What is the expected total time until the last of the three customers leaves?

   (c) What is the probability that Carol is the last one to leave?

5. Consider two independent Poisson processes $N_1(t)$ and $N_2(t)$ with respective rates λ_1 and λ_2. What is the probability that the two-dimensional process $(N_1(t), N_2(t))$ ever visits the point (i, j)?

6. **Required for APPM 5560 only** Events occur according to a Poisson process with rate λ. Each time an event occurs, we must decide whether or not to stop, with our objective being to stop at the last event to occur prior to some sepcified time T, where $T > 1/\lambda$. That is, if an event occurs at time t, $0 \leq t \leq T$, and we decide to stop, then we win if there are no additional events by time T, and we lose otherwise. If we do not stop when an event occurs and no additional events occur by time T, then we lose. Also, if no events occur by time T, then we lose. Consider the strategy that stops at the first event to occur after some fixed time s, $0 \leq s \leq T$.

   (a) Using this strategy, what is the probability of winning?

   (b) What value of s maximizes the probability of winning?

   (c) Show that one's probability of winning when using this strategy with the value of s you specified in part (b) is $1/e$.