1 Atkinson Chapter 4, problem 16. FYI this is an example of a 'Rodrigues formula' for orthogonal polynomials.

2 Let $p(x) = \sum_{i=0}^{n} c_i \phi_i(x)$ be the optimal degree-n polynomial approximation of a function $f \in C[a, b]$ where $\phi_i(x)$ are ortho*normal* polynomials. Prove the following (for any weighted L^2 norm)

(a) Bessel's inequalty: $\|p\| \le \|f\|$.

(b) Parseval's relation: $\sum_{i=0}^{\infty} c_i^2 = \|f\|^2$.

3 Atkinson Chapter 4, problem 24.

4 Let $\{\phi_i(x)\}_0^n$ be a family of ortho*normal* polynomials with respect to a weighted L^2 inner product. Define the kernel

$$K(x, y) = \sum_{i=0}^{n} \phi_i(x)\phi_i(y).$$

Prove that the following formula defines the optimal weighted L^2 polynomial

$$p(x) = \int_a^b K(x, y)f(y)w(y)\mathrm{d}y.$$

5 Implement a rootfinding method of your choice to compute all 9 roots of the 9th order Legendre polynomial. Give your answers to 5 decimal places.