APPM 5600: Homework #6
Due in class Wednesday November 1

1 We saw a discrete orthogonal Fourier basis in class, as the columns of the Vandermonde matrix for the equispaced trigonometric interpolation problem. Denote these orthogonal basis vectors by $\boldsymbol{v}^k$ with $k = 0, \ldots, 2n$; the j^{th} entry of the k^{th} vector is

$$\boldsymbol{v}_j^k = e^{\frac{2\pi \mathrm{i} jk}{2n+1}}$$

where $\mathrm{i} = \sqrt{-1}$.

A 'circulant' matrix of size$(2n+1) \times (2n+1)$ has the form

$$\mathbf{C} = \begin{bmatrix} a_0 & a_1 & \cdots & \cdots & a_{2n} \\ a_{2n} & a_0 & a_1 & \cdots & a_{2n-1} \\ a_{2n-1} & a_{2n} & a_0 & & a_{2n-2} \\ \vdots & & \ddots & \ddots & \vdots \\ a_1 & a_2 & \cdots & a_{2n} & a_0 \end{bmatrix}.$$

Let $\mathbf{S}$ be the matrix that shifts the index of a vector by 1, i.e. for any vector $\boldsymbol{w}$, the j^{th} entry of $\mathbf{S}\boldsymbol{w}$ is

$$(\mathbf{S}\boldsymbol{w})_j = w_{j+1}, \;\; j = 0, \cdots, 2n-1; \;\; (\mathbf{S}\boldsymbol{w})_{2n} = \boldsymbol{w}_0$$

(a) Show that any circulant matrix can be written as a polynomial of the $\mathbf{S}$ matrix.

(b) Prove that the vectors $\boldsymbol{v}^k$ are all eigenvectors of the circulant matrix. What are the eigenvalues?

2 Atkinson Chapter 3, problem 41.

3 Atkinson Chapter 4, problem 5.

4 Atkinson Chapter 4, problem 9.