APPM 5600: Homework #5
Due in class Wednesday October 25

1 Atkinson Chapter 3, problem 26.

2 Atkinson Chapter 3, problem 30.

3 Consider the vector space of cubic splines with knots $\{-1, 0, 1\}$. It's clear from the definition that any element of the space can be written as a piecewise polynomial of the form $a_0+b_0x+c_0x^2+d_0x^3$ on $[-1, 0]$ and $a_1+b_1x+c_1x^2+d_1x^3$ on $[0, 1]$. There are 8 coefficients in this representation, but they are not all free, because of the matching/smoothness conditions at $x = 0$. It was claimed during the proof that the space of splines has dimension $k+n-1$ (k^{th} order, $n+1$ knots) that the set $\{1, x, \ldots, x^{k-1}, (x-x_1)_+^{k-1}, \cdots, (x-x_{n-1})_+^{k-1}\}$ is a basis for the space of splines. Show explicitly that the set $\{1, x, x^2, x^3, (x-0)_+^3\}$ is a basis for the space of cubic splines with knots $-1, 0,$ and 1.

4 Consider interpolating the function $f(x) = x/(1 + 25x^2)$ over the interval $[-1, 1]$. Construct and plot [using a computer, not by hand] the function itself and (a) the following interpolants and (b) the absolute errors for the following

- The standard polynomial interpolant using 21 equispaced points
- The Hermite interpolant using 11 equispaced points.
- The piecewise-linear, continuous interpolant using 21 equispaced points (20 intervals)
- The piecewise-cubic Hermite interpolant using 11 equispaced points (10 intervals)
- The natural cubic spline interpolant using 21 equispaced points