APPM 5470, Methods of Applied Mathematics: Partial Differential Equations and Integral Equations, HW 4, due 10/20 *Fall 2017*

Book Problems:

Chapter 5: 6 (Heaviside function: $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x > 0$), 7 (assume appropriate decay of u), 9, 10

Additional Problems:

A1 Consider the inhomogeneous heat equation in the quarter plane

$$\begin{aligned} &u_t = u_{xx} + F(x,t), \quad x > 0, \quad t > 0, \\ &u(x,0) = 0, \quad x > 0, \\ &u(0,t) = 0, \quad t > 0, \end{aligned}$$

where $F \in C_1^2((0,\infty)^2)$, $F(0,t) = 0$, $F_{xx}(0,t) = 0$, $t > 0$ and F, F_t, and F_{xx} are bounded.

(a) Solve the initial-boundary value problem.

(b) Prove that the proposed solution satisfies the PDE in its appropriate domain, satisfies the initial condition, and satisfies the boundary condition. Rigorously justify every limit taken.

A2 In this problem, you will study the nonlinear, viscous Burgers equation

$$\begin{aligned} &u_t + uu_x = u_{xx}, \quad x \in \mathbb{R}, \quad t > 0, \\ &u(x,0) = f(x), \quad x \in \mathbb{R}. \end{aligned} \tag{1}$$

(a) Show that if $u(x,t) = -2\phi_x(x,t)/\phi(x,t)$, then $\phi(x,t)$ satisfies the *linear* heat equation, $\phi_t = \phi_{xx}$! This is known as the Hopf-Cole transformation.

(b) Use the Hopf-Cole transformation and the solution to the heat equation to provide a (formal) solution to the IVP in Eq. (1).