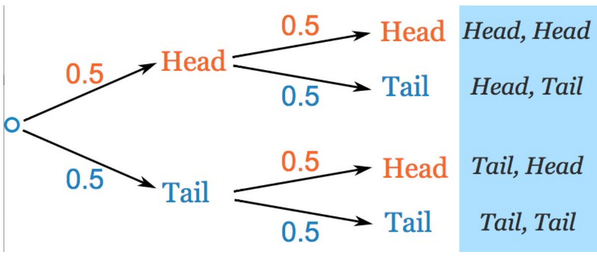
APPM 3570: Homework Set 3

Due Wed. Feb. 7, 2018

1. Chapter 2 in Ross: Problems 16, 25, 30, 42; Theoretical Exercise 18

2. Chapter 3 in Ross: Problems 9, 17

3. ***Win/loss tree diagram.*** In a best-of-three hockey tournament, the Penguins win their first game with probability 1/2. In each subsequent game, their probability of winning is $P(W_n|W_{n-1}) = 2/3$ if they win the previous game and $P(W_n|L_{n-1}) = 1/3$ if they lose the previous game. They stop playing either (i) when they win twice or (ii) when they have played three games, whichever comes first.



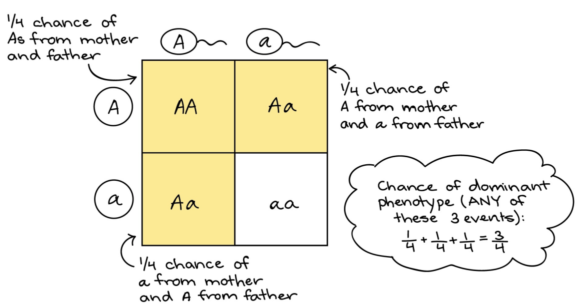
(a) Draw a tree diagram of their win/loss sequences. An *example* of a tree diagram for two coin flips is given above. Your tree diagram should represent all possible outcomes for the Penguins.

(b) The terminal vertices of your tree diagram constitute all the outcomes in the sample space S of the experiment wherein the Penguins play in the tournament. What is the sample space S?

(c) Define the set of all outcomes T wherein the Penguins win the tournament. Define the set of all outcomes F wherein the Penguins win the first game. Define the set of all outcomes R wherein the Penguins win the first game and win the tournament.

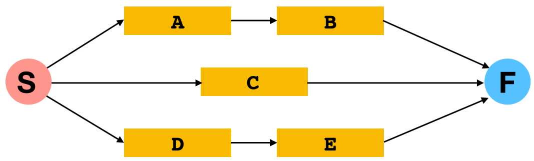
(d) Determine the probability of all the events listed in part (c).

(e) Compute the probability that the Penguins win the tournament, given that they win the first game.



4. ***Mendelian genetics.*** In diploid sexual reproduction, offspring get one of each gene alleles from each parent. Assume an equal probability of getting either allele from a parent. For instance, if the father has alleles Aa for a gene and the mother has alleles Aa, the offspring is AA with probability 1/4, Aa with probability 1/2, and aa with probability 1/4, as shown in the Punnett square above.

   The Hardy-Weinberg principle states that genotype frequencies in a population tend to a constant equilibrium, in the absence of other evolutionary influences. Show the probabilities $P(AA) = 1/4$, $P(Aa) = 1/2$ and $P(aa) = 1/4$ are such an equilibrium. That is, assuming both sexes of parents obey these probabilities in generation n ($P(AA_n) = 1/4$, $P(Aa_n) = 1/2$ and $P(aa_n) = 1/4$), their children's generation $(n+1)$ will have these probabilities too ($P(AA_{n+1}) = 1/4$, $P(Aa_{n+1}) = 1/2$ and $P(aa_{n+1}) = 1/4$). All parents randomly select mates.



5. ***System design.*** A common problem in engineering design is determining the most effective way to construct a system made of several potentially faulty components. In the above, road engineers have constructed five bridges (A,B,C,D,E) that allow a driver to get from Start to Finish three different ways (as shown above). Over the course of a year, each bridge has an equal probability of failing. If one bridge on a route fails, the route is impassable (e.g., if bridge A fails, the top route is impassable).
   (a) If three bridges fail, what is the probability all routes are impassable?
   (b) If four bridges fail, what is the probability all routes are impassable?
   (c) If all routes are known to be impassable, and it is known that exactly three bridges have failed, what is the probability that bridge B has failed?