2. ASSIGNMENT 2

Due Wednesday, February 7

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(1) The power method.

(a) Show that the Hilbert matrix (see below) is positive definite. Hint: use
$$\frac{1}{i+j-1} = \int_0^1 x^{i+j-2}\,dx\,.$$

(b) Implement the power method for finding the dominant eigenvalue λ_1, $|\lambda_1| > |\lambda_2| \geq |\lambda_3| \geq \cdots \geq |\lambda_n|$. Use the power method to find the dominant eigenvalue and the corresponding eigenvector for the Hilbert matrix,
$$A_{i,j} = \frac{1}{i+j-1},$$
where $i, j = 1, 2, \ldots, n$. The dominant eigenvalue is well-separated in this case.

(c) Use the power method (with the necessary modifications) to find the smallest eigenvalue of the Hilbert matrix with $n = 16$. How accurate is this eigenvalue? Is this consistent with the estimate $\min_{\lambda \in \sigma(A)} |\lambda - \mu| \leq \|E\|_2$, where μ is the eigenvalue of the perturbed matrix $A + E$?

(d) Assume that a real symmetric matrix A has eigenvalues $\lambda_1 = -\lambda_2$ and $|\lambda_1| = |\lambda_2| > |\lambda_3| \geq \cdots \geq |\lambda_n|$. Suggest a modification of the power method to find corresponding eigenvectors.

(e) A real symmetric matrix A has an eigenvalue 1 of multiplicity 8; the rest of the eigenvalues are ≤ 0.1 in absolute value. Describe an algorithm, based on the power method, for finding an orthogonal basis of the 8-dimensional eigenspace corresponding to the dominant eigenvalue. Estimate the number of necessary iterations to achieve double precision accuracy.