APPM 4/5560

Problem Set One (Due Wednesday, January 23)

Welcome to Problem Set One. Undergrads must do problems 1-5 and grads must do problem 1-6*.

Probability Pre-req Practice:

1. A fair coin is tossed until the same side appears twice in succession. Let N be the number of tosses required.

    (a) Find the pmf (probability mass function) for N.

    (b) Let A be the event that N is even and let B be the event that $N \leq 6$. Find $P(A)$, $P(B)$, and $P(A \cap B)$.

2. A gambler has, in his pocket, a fair coin and a two-headed coin. He selects one of the coins at random. When he flips it, it shows "heads". What is the probability that it is the fair coin?

New Stuff:

3. (Durrett 1.1) A fair coin is tossed repeatedly with results $Y_0, Y_1, Y_2, \ldots$ that are 0 or 1 with probability 1/2 each. For $n \geq 1$, let $X_n = Y_n + Y_{n-1}$ be the number of 1's in the $(n-1)$th and nth tosses. Is X_n a Markov chain? Justify your answer.

4. (Durrett 1.2) Five white balls and five black balls are distributed in two urns, labeled urn I and urn II, in such a way that each urn contains five balls. At each step we draw one ball from each urn and exchange them. Let X_n be the number of white balls in urn I at time n. Write down the transition probability matrix for X_n.

5. A Markov chain $\{X_n\}_{n=0}^{\infty}$ on the state space $S = \{0, 1, 2\}$ has transition probability matrix

$$
\mathbf{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \left\| \begin{array}{ccc} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0 & 0 & 1 \end{array} \right\| \end{array}.
$$

State 2 is called an "absorbing state" because, once the chain hits state 2 it will remain there forever!

    (a) What is $P(X_{10} = 2 | X_9 = 1)$?
    (b) What is $P(X_2 = 1 | X_0 = 1)$?
    (c) What is $P(X_3 \neq 2 | X_0 = 0)$?

Grad Problem:

6. Let $\{X_n\}$ be a Markov chain. While the value of X_2 may be dependent on the value of X_0, they are **conditionally independent** once you are given the value of X_1. Prove this!