

APPM 5600: Homework #1
Due in class Friday Sept 15

1 Prove that the condition number of a nonsingular matrix is greater than 1.

2 (a) Prove the following matrix identity (Sherman-Morrison).

$$(\mathbf{A} + \mathbf{u}\mathbf{v}^T)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{u}\mathbf{v}^T\mathbf{A}^{-1}}{1 + \mathbf{v}^T\mathbf{A}^{-1}\mathbf{u}}.$$

(b) Suppose that the LU factorization of \mathbf{A} is available, e.g. because you computed it. Explain how the Sherman-Morrison identity can be used to solve the system $(\mathbf{A} + \mathbf{u}\mathbf{v}^T)\mathbf{x} = \mathbf{b}$.

3 Theorem 8.2 in Atkinson states that Gaussian Elimination applied to a tridiagonal matrix satisfying certain diagonal-dominance conditions does not require pivoting. What is the operation count (give an exact formula) when applying Gaussian Elimination to a tridiagonal system without pivoting?

4 Problem #2 from Atkinson, chapter 8.