APPM 5470, Methods of Applied Mathematics: Partial Differential Equations and Integral Equations, HW 1, due 9/11 *Fall 2017*

Book Problems:

Chapter 1: problems 4, 7, 8

Chapter 2: problems 1, 5, 6

Additional Problem:

A1 Consider the initial/boundary value problem for the linear advection equation

$$\begin{aligned} u_t + u_x = 0, \quad (x,t) \in D = \{(x,t) \,|\, t > 0,\ x > bt\} \subset \mathbb{R}^2, \\ u(bt,t) = f(t) \in C^1(0,\infty), \quad t > 0, \qquad u(x,0) = g(x) \in C^1(0,\infty), \quad x > 0, \end{aligned} \tag{1}$$

where $0 \leq b < 1$ and $C^1(0,\infty)$ is the collection of all continuously differentiable functions with domain $(0,\infty)$. This is a **signaling problem** where a signal is emitted from a (possibly) moving source along $x - bt = 0$.

(a) Solve the initial/boundary value problem when $g(x) = 0$.

(b) Solve the initial/boundary value problem when $f(t) = 0$.

(c) Solve the full initial/boundary value problem.

(d) What happens if $b \geq 1$?