# APPM 3570: Homework Set 14

Due Wed. May 2

1. Chapter 7 in Ross: Problems 14, 17, 24, 36, 38, 45, 56, 58

2. **Gambling strategies.** Consider a game in which the probability of winning and losing each play is $p$ and $1 - p$. You will explore the effects of two gambling strategies.

   (a) A gambler bets $1 each time they play the game. What are their expected winnings after 3 plays?

   (b) Another gambler starts their bet at $1, and doubles their bet each time they lose. If they win, they reset their bet to $1 on the next bet. What are their expected winnings after 3 plays?

   (c) For what values of $p$ will the gambler described part (b) have more expected winnings than the gambler from part (a) in 3 plays?

3. **Polling as sub-sampling.** Before you work this problem, read through Example 4c in Section 7.4 of Ross. We will explore the variability in estimating a candidate's favorability by polling. Consider a country with $N$ voters that is having an upcoming election. Each person has an opinion $v_i$ which can have one of three values: 1 if they favor candidate A, 0 if they favor candidate B, and 1/2 if they are undecided. There are $n_A = p_A \cdot N$ people in favor of candidate A; $n_B = p_B \cdot N$ people in favor of candidate B; and $n_U = p_U \cdot N$ undecided (Note: $n_A + n_B + n_U = N$ and $p_A + p_B + p_U = 1$).

   (a) Select $n$ people uniformly randomly and define $I_i$ as an indicator variable that is 1 if person $i = 1, ..., N$ is sampled and 0 otherwise. Compute $E[I_i]$, $E[I_i, I_j]$, $Var(I_i)$, and $Cov(I_i, I_j)$.

   (b) Compute the correlation coefficient $\rho(I_i, I_j)$. How does it change as $n$ is increased?

   (c) Define $S = \sum_{i=1}^{N} v_i I_i$ as the total favorability obtained from the sample of $n$ people. Compute $E[S]$ and $Var[S]$. Note, it will not be quite the same as the result on p.310, since we have specified $v_i$ differently here.

   (d) For the proportional favorability $S/n$, compute $E[S/n]$ and $Var[S/n]$.

   (e) What happens to $Var[S/n]$ as $n$ is increased? Why? Also, explain what happens when $n = N$.

***Extra Credit: Correctly complete this problem to add 5 points back to your score on Exam 2.***

4. Consider two i.i.d. uniform random variables $X \sim U(0, 1)$ and $Y \sim U(0, 1)$. What is the probability that $X < 1/2$ given $X < Y$?