APPM 4/5560

Problem Set Eleven (Due Wednesday, May 2nd)

1. Use the Metropolis-Hastings algorithm to simulate many realizations of the random variable with density given by
$$f(x) = \frac{2}{9}xe^{-x^2/9}, \qquad x > 0.$$
Explore the convergence of your algorithm by giving histograms of your results with the true density superimposed using various "burn-in times".

2. For an M/M/1 queue in equilibrium with interarrival rate λ and service rate μ, find the expected length of typical "busy period" for the server. (A "busy period" is an uninterrupted sequence of services between server "idle periods".)

3. A very important "balance flow" equation in queueing theory is
$$L = \lambda W$$
where L is the mean queue length in equilibrium, λ is the customer arrival rate, and W is the average time spent by a customer in the system.

   Verify this balance equation for the M/M/1 queue by computing W "from scratch".

4. Consider an M/G/1 queue with arrival rate $\lambda = 1$ and uniform(0,1) service times.
   (a) What is the expected number of people to arrive during a typical service time?
   (b) What is the expected number of people in the system in equilibrium?

5. [**Required for 5560 Students Only**]
   (a) Prove that a continuous time birth and death process is transient if and only if
$$\sum_{n=1}^{\infty} \frac{\mu_1\mu_2\cdots\mu_n}{\lambda_1\lambda_2\cdots\lambda_n} < \infty.$$
   (b) Show that the $M/M/\infty$ queue is always recurrent.
   (c) When is the $M/M/1$ queue recurrent?