APPM 3570: Homework Set 10

Due Wed. Apr. 4, 2018

Note: To help out the LAs, please draw a grading table at the top of the first page of your homework. The table should have five rows and two columns, just like the ones drawn on your graded homework.

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1. Chapter 6 in Ross: Problems 1, 6, 9, 16, 23, 24; Theoretical Exercises 1, 5

2. **Arrival times.** Imagine you live in ancient times, before telephones. In each of the following, you plan to meet a friend, and your arrival times are independent random variables. For each situation, compute the probability you end up meeting each other.

   (a) Each of you arrives at your meeting spot at an independent uniformly distributed time between 8 and 9 pm, and wait for 20 minutes.

   (b) Each of you arrives at an independent exponentially distributed time (with rate $\lambda = 1/\text{hour}$) after 8pm, and waits 1 hour.

3. **Random cuts.** Define the following recursive process: Start with a stick of unit length, cut it at a location given by the uniform random variable $X_1 \sim U(0, 1)$. Then, take the left portion from $[0, X_1]$, and cut it at location given by $X_2 \sim U(0, X_1)$. Now, take that left portion, and cut it at a location $X_3 \sim U(0, X_2)$. Repeat this process indefinitely, each time drawing a random variable $X_{n+1} \sim U(0, X_n)$ for $n = 1, 2, 3, 4, 5, 6, ...$

   (a) Determine the joint distribution $f(x_1, x_2)$ of (X_1, X_2).

   (b) Use integration and your answer from part (a) to compute the p.d.f. $f_{x_2}(x_2)$ and the expectation $E[X_2]$ of the random variable X_2.

   (c) Compute the p.d.f. $f_{x_3}(x_3)$ and the expectation $E[X_3]$.

   Extra Credit

   (d) Prove $f_{x_n}(x_n) = (-1)^{n-1} \ln^{n-1}(x_n)/(n-1)!$ using induction. Then, use this formula to show that the expectation $E[X_n] = \frac{1}{2^n}$ for any n.