

APPM 3570/STAT 3100 Spring 2019 Homework 10 - Due Apr. 3

- Chapter 6, Problems 6, 8, 9, 13, 26, 27.
 - Chapter 6, Theoretical Exercise 3.
1. **Arrival times.** Imagine you live in ancient times, before cellphones. In each of the following, you plan to meet a friend, and your arrival times are independent random variables. For each situation, compute the probability you end up meeting each other.
 - (a) Each of you arrives at your meeting spot at an independent uniformly distributed time between 8 and 9 pm, and wait for 20 minutes.
 - (b) Each of you arrives at an independent exponentially distributed time (with rate $\lambda = 1/\text{hour}$) after 8pm, and waits 1 hour.
 2. Let X and Y be independent standard normal random variables, and $Z = X/Y$.
 - (a) Find the CDF of Z , $F(z) = P(\frac{X}{Y} \leq z)$. The answer will be in terms of definite integrals. You might need to separate the cases $Y > 0$ and $Y < 0$.
 - (b) Find the PDF of Z , $f(z)$. The answer will be a simple function of z .
 3. **Random cuts.** Define the following recursive process: Start with a stick of unit length, cut it at a location given by the uniform random variable $X_1 \sim U(0, 1)$ (the notation $X \sim U(a, b)$ means the random variable X is uniformly distributed in $[a, b]$). Then, take the left portion from $[0, X_1]$, and cut it at location given by $X_2 \sim U(0, X_1)$. Now, take that left portion, and cut it at a location $X_3 \sim U(0, X_2)$. Repeat this process indefinitely, each time drawing a random variable $X_{n+1} \sim U(0, X_n)$ for $n = 1, 2, 3, 4, 5, 6, \dots$.
 - (a) Determine the joint distribution $f(x_1, x_2)$ of (X_1, X_2) .
 - (b) Use integration and your answer from part (a) to compute the p.d.f. $f_{x_2}(x_2)$ and the expectation $E[X_2]$ of the random variable X_2 .
 - (c) Compute the p.d.f. $f_{x_3}(x_3)$ and the expectation $E[X_3]$.

Extra Credit

 - (d) Prove $f_{x_n}(x_n) = (-1)^{n-1} \ln^{n-1}(x_n)/(n-1)!$ using induction. Then, use this formula to show that the expectation $E[X_n] = \frac{1}{2^n}$ for any n .

Note, as with all homework sets in this class, that you may discuss the homework problems with your classmates. However, the work you turn in must be your own – you should write your solutions on your own. Identical solutions will be considered as a violation of the Student Honor Code. Furthermore, **no work equals no credit**. Your homework should be neatly written or typed and stapled.

On the front of your homework clearly print your:

- First Name and Last Name
- Lecture number (either Section 001 or Section 002) and homework number.
- Draw a **blank grading table** with room for 3 problems, format points and a total:
- Points will be deducted if these instructions are not followed.

Remember that writing style, clarity, and completeness of explanations are always important. Justify your answers.