10. ASSIGNMENT 10

Due Wednesday, April 25

Gregory Beylkin, ECOT 323

(1) Solve the Poisson's equation
$$\Delta u = f$$
on the square $(x,y) : \ 0 \le x, y \le 1$ with the homogeneous Dirichlet boundary conditions. Assume that the function f is well approximated by
$$f(x,y) = \sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_{mn} \sin(\pi m x) \sin(\pi n y).$$

Choose appropriate discretization for $f(x,y)$ and organize computation to use the Fast Fourier Transform. Verify your results on several examples.

(2) Let ∂D be the ellipse $x^2/a^2 + y^2/b^2 = 1$. Consider the boundary value problem
$$\Delta u = 1, \quad (x,y) \in D,$$

$$u = x^4 + y^4, \quad (x,y) \in \partial D.$$

  (a) Reduce the problem to that with homogeneous boundary conditions

  (b) Reduce the problem to the Dirichlet problem for the Laplace equation.