3. Assignment 3

Due Wednesday, February 14

Gregory Beylkin, ECOT 323

(1) Prove that

- (a) if all singular values of matrix $A \in C^{n \times n}$ are equal, then $A = \gamma U$, where U is a unitary matrix and γ is a constant
- (b) if $A \in C^{n \times n}$ is nonsingular and λ is an eigenvalue of A, then $||A^{-1}||_2^{-1} \le |\lambda| \le ||A||_2$ (c) Show that any square matrix $A \in C^{n \times n}$ may be represented in the form A = SU, where S is a hermitian nonnegative definite matrix and U is a unitary matrix. Show that if A is invertible such representation is unique.
- (2) Atkinson, Problem 6 page 649. (Prove a perturbation estimate for the eigenvector corresponding to a simple eigenvalue on page 600. Read a hint on page 649/650).