Mortality and Healthcare

An Analysis Under Epstein-Zin Preferences

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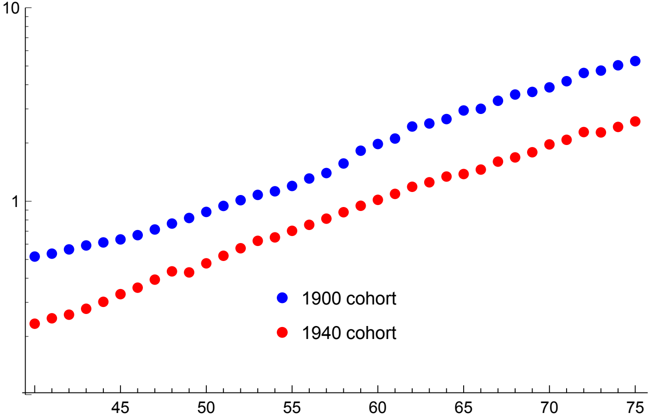
This is research on **DEATH**

- How to make life more pleasant?
  - **consumption**: feels good at the moment.
  - **investment**: enlarges wealth to sustain future consumption.
  - **healthcare**: defers death.

MORTALITY V.S. AGE

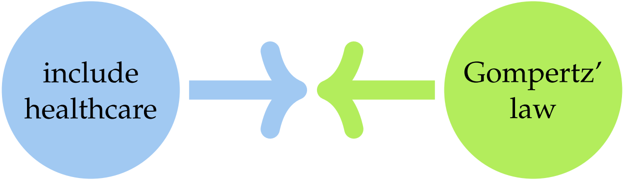


- Exponential increase in age [**Gompertz' law**]:

$$dM_t = \beta M_t dt \qquad (\beta \approx 7.1\%)$$

LITERATURE

- How Exogenous Mortality affects Consumption?
  - **Yarri (1965), Richard (1975), Davidoff et al (2005)**
  - Healthcare?
- Health as Capital, Healthcare as Investment
  - **Grossman (1972), Ehrlich and Chuma (1990)**
  - Health Capital observable?
- Mortality rates decline with health capital.
  - **Ehrlich (2000), Ehrlich and Yin (2005), Yogo (2009), Hugonnier et al. (2012)**
  - Gompertz' law?



PROBLEM FORMULATION

- Individual maximizes utility from lifetime consumption:

$$\sup_{c,\pi,h} \mathbb{E}\left[\int_0^\tau e^{-\delta t} U(c_t X_t)dt + U(\zeta X_\tau)\right].$$

- Money can buy...
  - consumption, which generates utility...
  - healthcare, which reduces mortality growth...
    $\Longrightarrow$ buying time for more consumption.
  - investment, which potentially enlarges wealth X...
- $\zeta \in (0, 1)$: Inheritance and estate taxes.

QUESTIONS

- Find optimal controls $\{\hat{c}_t\}_{t\geq 0}$, $\{\hat{\pi}_t\}_{t\geq 0}$, $\{\hat{h}_t\}_{t\geq 0}$.
- $\{\hat{h}_t\}_{t\geq 0} \Longrightarrow$ *endogenous* mortality curve
  $\Longrightarrow$ follows Gompertz' law?

Utility Function

Isoelastic utility:

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \qquad 0 < \gamma \neq 1.$$

- γ: risk aversion
- $1/\gamma$: elasticity of inter-temporal substitution (EIS).
  - EIS large $\implies$ substitute *future* consumption for *current* consumption.
  - EIS small $\implies$ substitute *current* consumption for *future* consumption.

MORTALITY DYNAMICS

- Without healthcare, mortality grows exponentially **[Gompertz' law]**:

$$dM_t = \beta M_t dt.$$

- Healthcare slows down mortality growth

$$dM_t = (\beta - g(h_t))M_t dt$$

  - h_t: healthcare-wealth ratio
  - $g : \mathbb{R}_+ \to \mathbb{R}_+$ measures *efficacy* of healthcare
    - $g(0) = 0$, g is increasing and concave.
    - Assume $g \in C^2$, $g'(0) = \infty$, and $g'(\infty) = 0$ (Inada's condition).
    - Example:

$$g(h) = \frac{a}{q} h^q \qquad a > 0, q \in (0, 1)$$

WEALTH DYNAMICS

- Consider a Black-Scholes market with
  - a riskfree rate $r > 0$;
  - a risky asset

$$dS_t = (\mu + r)S_t dt + \sigma S_t dB_t,$$

    for some $\mu \in \mathbb{R}$ and $\sigma > 0$.
- The wealth process evolves as

$$dX_t = (r - c_t - h_t + \mu\pi_t)X_t dt + \sigma\pi_t X_t dB_t. \qquad (1)$$

PREVIOUS RESULTS

- **Guasoni & Huang (2019)** analyze the value function

$$V(x,m) := \sup_{c,\pi,h} \mathbb{E}\left[\int_0^\tau e^{-\delta t} U(c_t X_t)dt + U(\zeta X_\tau)\right]$$

  - $V(x,m)$ is solved semi-explicitly.
  - $\{\hat{c}_t\}$, $\{\hat{\pi}_t\}$, $\{\hat{h}_t\}$ are characterized as functions of $V(x,m)$.

- **Calibration Issue:**

  Should $\gamma > 0$ be calibrated to risk aversion or EIS?

  Bansal & Yaron (2004): risk aversion and EIS are both larger than 1.

  - Risk aversion (γ) $> 1 \implies \gamma > 1$.
  - EIS ($1/\gamma$) $> 1 \implies \gamma < 1$.

This Paper

Given $(c_t, h_t)_{t\geq 0}$, define Epstein-Zin utility process on the random horizon τ as the semimartingale $(\widetilde{V}_t^{c,h})_{t\geq 0}$ satisfying

$$\widetilde{V}_t^{c,h} = \mathbb{E}\left[\int_{t\wedge\tau}^{T\wedge\tau} f(c_s, \widetilde{V}_s^{c,h})ds + \zeta^{1-\gamma}\widetilde{V}_{\tau-}^{c,h}\mathbb{1}_{\{\tau\leq T\}} + \widetilde{V}_T^{c,h}\mathbb{1}_{\{\tau>T\}}\middle|\mathcal{G}_t\right], \forall t \leq T < \infty. \tag{2}$$

- Epstein-Zin aggregator:

$$f(c,v) := \delta\frac{c^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}\left((1-\gamma)v\right)^{1-\frac{1}{\theta}} - \delta\theta v, \quad \text{with } \theta := \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

  - EIS: ψ, risk aversion: γ.
    (Duffie & Epstein (1992a), Duffie & Epstein (1992b))
  - Assume $\psi > 1$ and $\gamma > 1$.

BSDE Characterization

Given $(c, h)_{t\geq 0}$, a semimartingale $\widetilde{V}$ solves (2) *if and only if*

$$\widetilde{V}_t = V_t \mathbb{1}_{\{t<\tau\}} + \zeta^{1-\gamma} V_{\tau-} \mathbb{1}_{\{t\geq\tau\}} \qquad \forall t \geq 0, \tag{3}$$

where V is a semimartingale solving the infinite-horizon BSDE

$$dV_t = -F(c_t, M_t^h, V_t)ds + d\mathscr{M}_t, \quad \forall\, 0 \leq t \leq T < \infty, \tag{4}$$

with $F : \Omega \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ defined by

$$F(c, m, v) := f(c, v) - (1 - \zeta^{1-\gamma})mv. \tag{5}$$

- For BSDE (4), *uniqueness* of solutions is challenging.
  $\Longrightarrow$ need to focus on a special class of $(c_t, h_t)_{t\geq 0}$.

Definition

Fix $k \in \mathbb{R}$. We say $(c_t, h_t)_{t\geq 0}$ is k-admissible if there exists a solution V to BSDE (4) satisfying

- Integrability: $\mathbb{E}\left[\sup_{s\in[0,t]} |V_s|\right] < \infty \quad \forall t > 0$;
- Transversality:

$$\lim_{t\to\infty} e^{-(\delta\theta+(1-\theta)k)t}\mathbb{E}\left[e^{-\gamma(\psi-1)\frac{1-\zeta^{1-\gamma}}{1-\gamma}\int_0^t M_s^h ds}|V_t|\right] = 0; \qquad (6)$$

- Boundedness from above by a tractable process:

$$V_s \leq \delta^\theta \left(k + (\psi-1)\frac{1-\zeta^{1-\gamma}}{1-\gamma}M_s^h\right)^{-\theta} \frac{c_s^{1-\gamma}}{1-\gamma}, \quad \forall s \geq 0.$$

- Find k that is "just right":
  - Small $k \implies$ Strong transversality, boundedness conditions
  - Large $k \implies$ Weak transversality, boundedness conditions

Theorem

Fix $k \in \mathbb{R}$. For any k-admissible (c, h), there exists a unique solution $V^{c,h}$ to BSDE (4). Hence, the Epstein-Zin utility process $\widetilde{V}^{c,h}$ in (2) can be uniquely determined via

$$\widetilde{V}_t^{c,h} = V_t^{c,h} \mathbb{1}_{\{t<\tau\}} + \zeta^{1-\gamma} V_{\tau-}^{c,h} \mathbb{1}_{\{t\geq\tau\}} \qquad \forall t \geq 0.$$

The Control Problem

An agent maximizes her Epstein-Zin utility $\widetilde{V}_0^{c,h}$ by choosing (c, π, h) in a suitable collection of strategies $\mathcal{P}$, i.e.

$$\sup_{(c,\pi,h)\in\mathcal{P}} \widetilde{V}_0^{c,h} = \sup_{(c,\pi,h)\in\mathcal{P}} V_0^{c,h}.$$

The set $\mathcal{P}$ contains (c, π, h) satisfying

- (c, h) is k-admissible for some $k \in \mathbb{R}$,
- π is s.t. wealth process $X^{c,\pi,h}$ in (1) well-defined.

PDE Characterization

- Under the current Markovian framework, we take

$$v(x,m) := \sup_{(c,\pi,h)\in\mathcal{P}} \widetilde{V}_0^{c,h}. \tag{7}$$

- Take $k \in \mathbb{R}$ (encoded implicitly in $\mathcal{P}$) to be

$$k^* := \delta\psi + (1-\psi)\left(r + \frac{1}{2\gamma}\left(\frac{\mu}{\sigma}\right)^2\right),$$

which is the optimal consumption rate of an immortal agent ($m = 0$).

Theorem

Assume $g\left((g')^{-1}(\psi-1)\right) < \beta$ *and* $\delta\psi + (1-\psi)\left(r + \frac{1}{2\gamma}\left(\frac{\mu}{\sigma}\right)^2\right) > 0$. *Then,*

$$v(x,m) = \delta^{\theta}\frac{x^{1-\gamma}}{1-\gamma}u^*(m)^{-\frac{\theta}{\psi}}, \quad (x,m) \in \mathbb{R}_+^2,$$

where $u^* : \mathbb{R}_+ \to \mathbb{R}_+$ *is the unique nonnegative, strictly increasing, strictly concave, classical solution to*

$$\begin{aligned} 0 = u(m)^2 - \tilde{c}_0(m)u(m) - \beta m u'(m) \\ + m u'(m) \sup_{h \in \mathbb{R}_+} \left\{ g(h) - (\psi - 1)\frac{u(m)}{m u'(m)} h \right\}, \end{aligned}$$

where $\tilde{c}_0(m) := \psi\delta + (1-\psi)\left(\frac{(\zeta^{1-\gamma}-1)m}{1-\gamma} + r + \frac{1}{2\gamma}\left(\frac{\mu}{\sigma}\right)^2\right)$. *Furthermore,*

$$\hat{c}_t := u^*(M_t), \quad \hat{\pi}_t := \frac{\mu}{\gamma\sigma^2}, \quad \hat{h}_t := (g')^{-1}\left((\psi-1)\frac{u^*(M_t)}{M_t (u^*)'(M_t)}\right), \quad t \geq 0$$

is an optimal control for $\sup_{(c,\pi,h)\in\mathcal{P}} V_0^{c,h}$.

Remarks

- **Conditions of Theorem:**
  1. $g\left((g')^{-1}(\psi - 1)\right) < \beta \implies g(\hat{h}_t) < \beta \ \forall t \geq 0.$
     - Even optimal healthcare spending can only slow (but not reverse) aging.
  2. $\delta\psi + (1-\psi)\left(r + \frac{1}{2\gamma}\left(\frac{\mu}{\sigma}\right)^2\right) > 0 \implies \hat{c}_t > 0$ for all $t \geq 0$.
- **Proof Sketch:**
  1. Construction of u^*
     - No healthcare ($g \equiv 0$) $\implies$ supersolution $\bar{u}$
     - Forever young ($\beta = 0$) $\implies$ subsolution $\tilde{u}$
     - By *Perron's method* of viscosity solutions, construct

$$\tilde{u} \leq u^* \leq \bar{u}.$$

  2. Upgrade regularity

     "Concavity" $+$ viscosity solution $\implies$ smoothness.

Remarks

- **Proof Sketch (conti.):**
  3. Verification:
     - Classical arguments do not work....
     - Relate the candidate solution

$$w(x,m) := \delta^{\theta} \frac{x^{1-\gamma}}{1-\gamma} u^{*}(m)^{-\frac{\theta}{\psi}}$$

       to a BSDE.
     - Compare this BSDE for w with BSDE (4).
     - By a (new) *comparison principle* for *infinite-horizon BSDEs*, $w(x,m) = v(x,m)$.

CALIBRATION

- Parameters taken as given from empirical studies:

  $r = 1\%, \delta = 3\%, \psi = 1.5, \gamma = 2, \zeta = 50\%, \mu = 5.2\%, \sigma = 15.4\%.$

- The efficacy function $g : \mathbb{R}_+ \to \mathbb{R}_+$ is taken as

$$g(z) = a\frac{z^q}{q}, \quad \text{with } a > 0 \text{ and } q \in (0,1)$$

- Calibrate $\beta > 0, a > 0, q \in (0,1)$ to actual mortality data.
  - β: Estimated from mortality data for 1900 cohort (assuming no healthcare available).
  - a, q: Calibrated by minimizing mean square error between endogenous mortality curve and mortality data of 1940 cohort.

The US

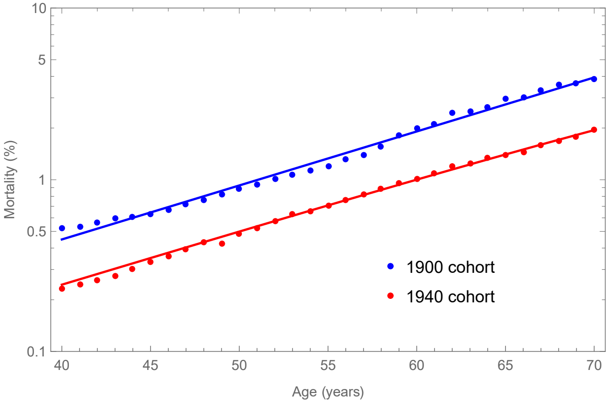
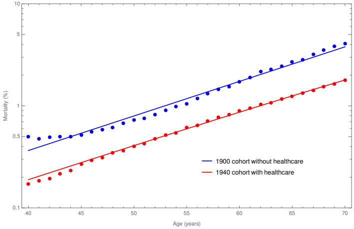
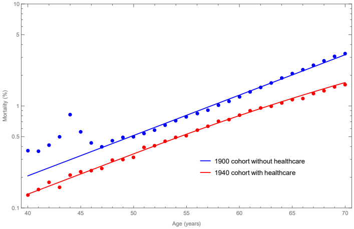


Figure: Mortality rates (vertical axis, in logarithmic scale) at adults' ages for the cohorts born in 1900 and 1940 in the US. The dots are actual mortality data (Source: Berkeley Human Mortality Database), and the lines are model-implied mortality curves.

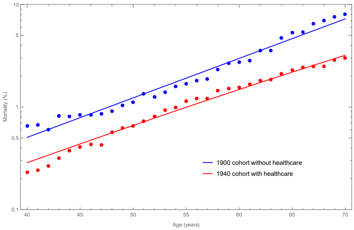
OTHER COUNTRIES



(a) UK



(b) Netherlands



(c) Bulgaria

Calibrated Efficacy

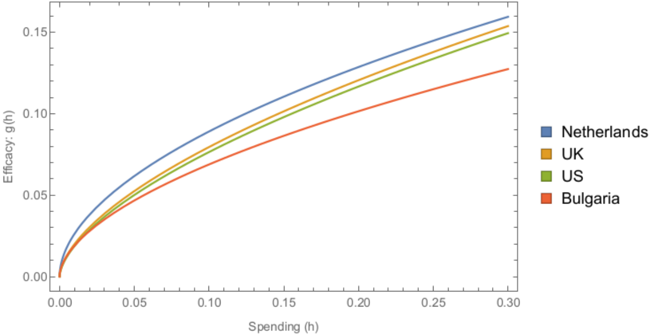


Figure: Calibrated efficacy of healthcare $g(h)$, measured by the reduction in the growth of mortality, given proportions of wealth h spent on healthcare in different countries.

- In line with WHO's ranking of healthcare systems.

HEALTHCARE SPENDING

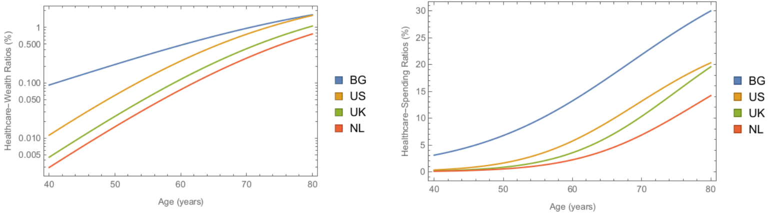


Figure: Optimal healthcare spending in the US, UK, Netherlands (NL), and Bulgaria (BG). Left panel: Healthcare-wealth ratio (vertical, log-scale) at adult ages (horizontal). Right panel: Healthcare as a fraction of total spending in consumption, investment, and healthcare (vertical) at adult ages (horizontal).

THANK YOU!!

Q & A

Preprint available @ https://arxiv.org/abs/2003.01783

"Mortality and Healthcare: a Stochastic Control Analysis under Epstein-Zin Preferences"