Healthcare and Consumption with Aging

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This is research on **DEATH**

- How to make life more pleasant?
  - **consumption**: feels good at the moment.
  - **healthcare**: defers death.

Do We Know Healthcare?

Journal of American Medical Association (JAMA)

- On July 11, 2016, Barack Obama published

  "United States Health Care Reform –Progress to Date and Next Steps"

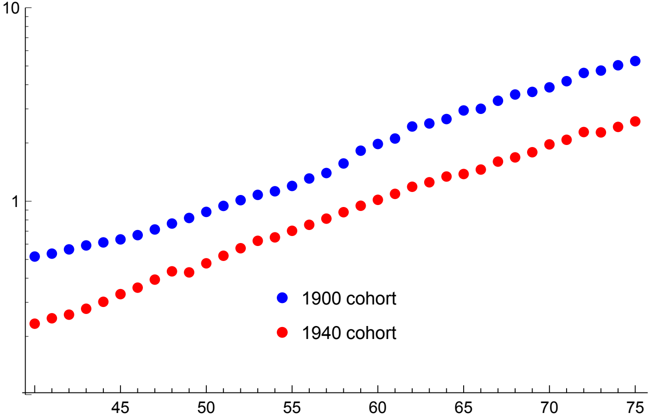
- Editorial summary:

  4 big surprises from ACA

  - e.g. cost of healthcare ↓ while quality ↑.

All pundits of healthcare were WRONG about ACA

Mortality v.s. Age

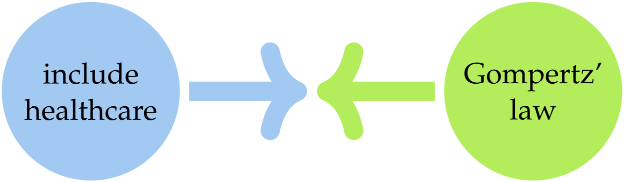


- Exponential increase in age **[Gompertz' law]**:

$$dM_t = \beta M_t dt \qquad (\beta \approx 7.1\%)$$

LITERATURE

- How Exogenous Mortality affects Consumption?
  - **Yarri (1965), Richard (1975), Davidoff et al (2005)**
  - Healthcare?
- Health as Capital, Healthcare as Investment
  - **Grossman (1972), Ehrlich and Chuma (1990)**
  - Health Capital observable?
- Mortality rates decline with health capital.
  - **Ehrlich (2000), Ehrlich and Yin (2005), Yogo (2009), Hugonnier et al. (2012)**
  - Gompertz' law?



THIS PAPER

IDEA

- Household maximizes utility from lifetime consumption:

$$\sup_{c,h} \mathbb{E}\left[\int_0^\tau e^{-\delta t} U(c_t X_t) dt\right].$$

- Money can buy...
  - consumption, which generates utility...
  - healthcare, which reduces mortality growth...

  $\Longrightarrow$ buying time for more consumption.

QUESTIONS

- Find optimal control processes

$$\{\hat{c}_t\}_{t\geq 0}, \quad \{\hat{h}_t\}_{t\geq 0}.$$

- $\{\hat{h}_t\}_{t\geq 0} \Longrightarrow$ *endogenous* mortality curve
  $\Longrightarrow$ follows Gompertz' law?

The Value Function

- Naïve approach:

$$\sup_{c,h} \mathbb{E}\left[\int_0^{\tau} e^{-\delta t} U(c_t X_t)\,dt\right].$$

- NOT invariant to utility translation!

  If U becomes $U + k$,

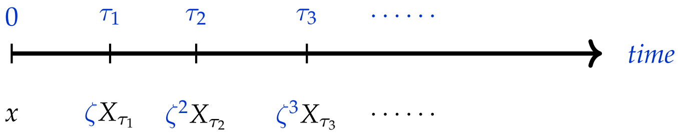
$$\sup_{c,h}\left\{\mathbb{E}\left[\int_0^{\tau} e^{-\delta t} U(c_t X_t)\,dt\right] + k\mathbb{E}\left[\frac{1-e^{-\delta\tau}}{\delta}\right]\right\}$$

OBSERVE:

τ is endogenous $\implies$ NO translation invariance.

THE VALUE FUNCTION

- Our approach:
  - After death, household carries on with the same optimization problem.



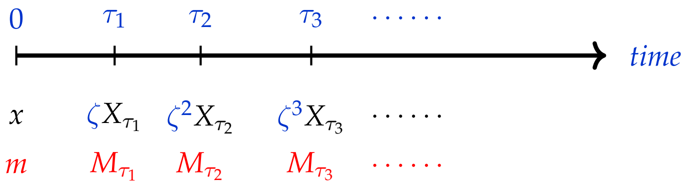
  - Death scales household wealth by factor $\zeta \in [0, 1]$. (ζ: inheritance tax, annuity loss, foregone income...)
- The value function:

$$V(x, m) = \sup_{c,h} \mathbb{E}\left[\sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt\right] \quad \text{with } \tau_0 := 0.$$

(Translation Invariant)

ASSUMPTIONS

- Simplifications:



  - Surviving spouse in similar age group.
  - Most weight carried by first two lifetimes.

- Isoelastic utility:

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \qquad 0 < \gamma \neq 1$$

Mortality Dynamics

- Without healthcare, mortality grows exponentially **[Gompertz' law]**:

$$dM_t = \beta M_t dt.$$

- Healthcare slows down mortality growth

$$dM_t = (\beta - g(h_t))M_t dt$$

  - h_t: healthcare-wealth ratio
  - $g : \mathbb{R}_+ \mapsto \mathbb{R}_+$ measures *efficacy* of healthcare
    - $g(0) = 0$, g is increasing and concave.
    - Example:

$$g(h) = \frac{a}{q} h^q \qquad a > 0, q \in (0, 1)$$

Assumptions

Efficacy depends on healthcare-wealth ratio.

- Means-tested subsidies;
- **Chetty et al. (2016, JAMA)**: life expectancy is significantly correlated with health behaviors but not with access to medical care.

$\Longrightarrow$ h_t reflects time and lost-income costs.

Wealth Dynamics

Household wealth grows at a constant interest rate $r > 0$, minus consumption and health spending:

$$dX_t = (r - c_t - h_t)X_t dt.$$

A Stochastic Control Problem

- The value function:

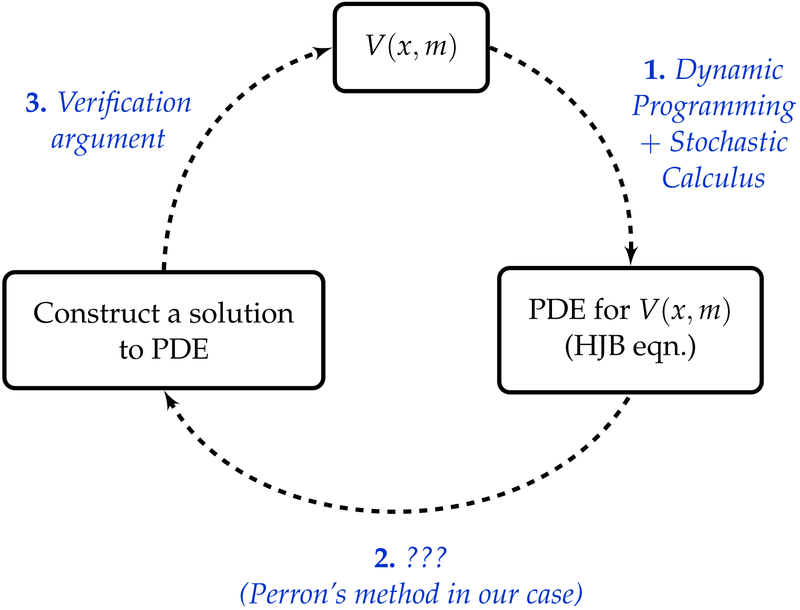
$$V(x,m) = \sup_{c,h} \mathbb{E}\left[\sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt\right]$$

- State processes:

$$\begin{aligned} dX_t &= (r - c_t - h_t) X_t dt \quad X_0 = x, \\ dM_t &= (\beta - g(h_t)) M_t dt, \quad M_0 = m. \end{aligned}$$

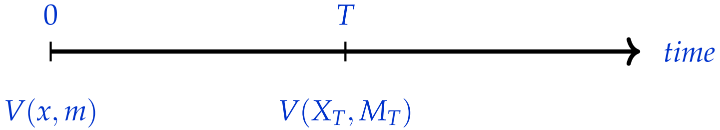
- Distributions of death times:

$$\mathbb{P}(\tau_n > t \mid \tau_{n-1} < t) = \exp\left\{-\int_{\tau_{n-1}}^{t} M_s ds\right\}.$$


$V(x,m)$
1. *Dynamic Programming + Stochastic Calculus*
PDE for $V(x,m)$ (HJB eqn.)
2. *???*
(Perron's method in our case)
Construct a solution to PDE
3. *Verification argument*

Dynamic Programming Principle

$$V(x,m) = \sup_{c,h} \mathbb{E}\left[\sum_{n=1}^{\infty} \int_{\tau_{n-1}}^{\tau_n} e^{-\delta t} U(\zeta^n X_t c_t) dt\right]$$



- DPP states:

$$V(x,m) = \sup_{c,h,\pi} \mathbb{E}\left[\int_0^T e^{-\int_0^t (\delta + M_s) ds} [U(c_t X_t) + M_t V(\zeta X_t, M_t)] dt \right.$$
$$\left. + e^{-\int_0^T (\delta + M_s) ds} V(X_T, M_T)\right].$$

Itô's Formula

- By Itô's formula in stochastic calculus,

$$
\begin{aligned}
&d\left(e^{-\int_0^t(\delta+M_s)ds}V(X_t,M_t)\right)\\
&\quad= e^{-\int_0^t(\delta+M_s)ds}\Big[-(M_t+\delta)V(X_t,M_t)+V_x(X_t,M_t)dX_t\\
&\qquad\qquad + V_m(X_t,M_t)dM_t+\frac{1}{2}V_{xx}(X_t,M_t)(dX_t)^2\Big].
\end{aligned}
$$

- "DPP + Itô's Formula" yields

$$
\begin{aligned}
0 = \sup_{c,h} \mathbb{E}\Bigg[\int_0^T e^{-\int_0^t (\delta + M_s) ds} \Bigg(& U(c_t X_t) + M_t V(\zeta X_t, M_t) \\
& - (\delta + M_t) V(X_t, M_t) + [r + \mu \pi_t - c_t - h_t] X_t V_x(X_t, M_t) \\
& + (\beta - g(h_t)) M_t V_m(X_t, M_t) + \frac{1}{2} \sigma^2 \pi^2 X_t^2 V_{xx}(X_t, M_t) \Bigg) dt \Bigg].
\end{aligned}
$$

- A Big Guess: $V(x, m)$ is a solution to the PDE

$$
\begin{aligned}
0 = \sup_{c,h \geq 0} \Big\{ & U(cx) + mV(\zeta x, m) - (\delta + m) V(x, m) \\
& + [r + \mu \pi - c - h] x V_x(x, m) + (\beta - g(h)) m V_m(x, m) \\
& + \frac{1}{2} \sigma^2 \pi^2 x^2 V_{xx}(x, m) \Big\}.
\end{aligned}
$$

HAMILTON-JACOBI-BELLMAN EQUATION

- The HJB equation for V:

$$\begin{aligned}
&\sup_{c\geq 0}\{U(cx) - hxV_x(x,m)\} \\
&+ \sup_{h\geq 0}\{-mg(h)V_m(x,m) - hxV_x(x,m)\} \\
&- \delta V(x,m) + rxV_x(x,m) \\
&+ (V(\zeta x,m) - V(x,m))m + \beta m V_m(x,m) = 0. \qquad \text{(pde)}
\end{aligned}$$

- Optimal strategies:

$$\hat{c} = \frac{V_x(x,m)^{-\frac{1}{\gamma}}}{x}, \quad \hat{h} = (g')^{-1}\left(\frac{-xV_x(x,m)}{mV_m(x,m)}\right)$$

REDUCTION TO ODE

- Taking $\boxed{V(x,m) = \frac{x^{1-\gamma}}{1-\gamma} u(m)^{-\gamma}}$ gives

$$u(m)^2 - \left(\frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r \right) u(m)$$
$$+ m u'(m) \left(\sup_{h \geq 0} \left\{ g(h) - \frac{1-\gamma}{\gamma} \frac{u(m)}{m u'(m)} h \right\} - \beta \right) = 0. \qquad \text{(ode)}$$

- Optimal strategies:

$$\hat{c} = \frac{V_x(x,m)^{-\frac{1}{\gamma}}}{x} = u(m),$$
$$\hat{h} = (g')^{-1} \left(\frac{-x V_x(x,m)}{m V_m(x,m)} \right) = (g')^{-1} \left(\frac{1-\gamma}{\gamma} \frac{u(m)}{m u'(m)} \right).$$

THREE SETTINGS

To understand effects of aging and healthcare, consider

1. **Forever Young** [Neither Aging nor Healthcare]:

$$M_t \equiv m \geq 0.$$

2. **Gompertz's law** [Aging without Healthcare]:

$$dM_t = \beta M_t dt, \quad M_0 = m > 0.$$

3. **The general case** [Aging with Healthcare]:

$$dM_t = (\beta - g(h_t))M_t dt, \quad M_0 = m > 0.$$

Neither Aging nor Healthcare

- $M_t \equiv m > 0$ (forever young).
- (ode) reduces to

$$u(m)^2 - \left(\frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r \right) u(m) = 0.$$

- Optimal strategy:

$$\hat{c} = u(m) = c_0(m) := \frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r.$$

Aging without Healthcare

- $dM_t = \beta M_t dt$.
- This is non-standard, cf. **Huang, Milevsky, & Salisbury (2012)**)
- (ode) reduces to

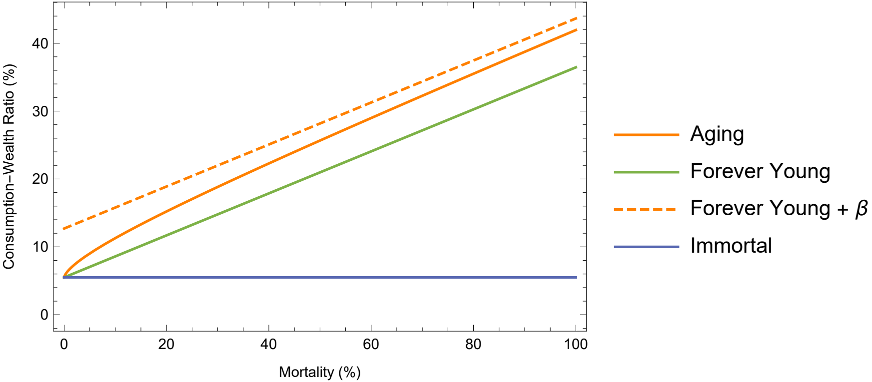
$$u(m)^2 - \left(\frac{\delta + (1-\zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r\right) u(m) - \beta m u'(m) = 0.$$

- Optimal strategy:

$$\begin{aligned}\hat{c} &= u(m) \\ &= c_\beta(m) := \left(\int_0^\infty e^{-\frac{(1-\zeta^{1-\gamma})my}{\gamma}} (\beta y + 1)^{-\left(1+\frac{\delta+(\gamma-1)r}{\beta\gamma}\right)} dy\right)^{-1}\end{aligned}$$

- Asymptotics for old age (large m):

$$c_\beta(m) = c_0(m) + \beta + O(\frac{1}{m}).$$



- green curve: c_0
- orange curve: c_β
- Mortality and aging have large impacts on $\hat{c}$.

Aging with Healthcare

- $dM_t = (\beta - g(h_t))M_t dt$.
- $\boxed{\textbf{Assumption}}$: healthcare...
  - *slows* aging,
  - *does not stop* aging.
- **Intuitive idea:** a solution u^* to (ode) should satisfy

$$c_0 \leq u^* \leq c_\beta, \quad u^* \text{ is concave.}$$

- **Observe:** Under the condition

$$\boxed{g\left((g')^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right) < \beta},$$

  - c_β is a supersolution to (ode),
  - c_0 is a subsolution to (ode).

CONSTRUCTION OF u^*

- **Perron's method:**

$$u^*(m) := \inf_{u \in \mathcal{S}} u(m) \quad m > 0,$$

where $\mathcal{S}$ is the collection of $u : \mathbb{R}_+ \mapsto \mathbb{R}_+$ satisfying

  - $c_0 \leq u \leq c_\beta$,
  - u is a viscosity supersolution to (ode).
  - u is concave, increasing.

$$\implies \quad u^* \text{ is a viscosity solution to (ode)}$$

- **Regularity:**

$$\underline{\text{viscosity solution property}} + \underline{\text{concavity}}$$

$$\implies u^* \text{ is a classical solution to (ode)}$$

Verification

- $u^*(m)$ is a classical solution to (ode).
- $\frac{x^{1-\gamma}}{1-\gamma}(u^*(m))^{-\gamma}$ is a classical solution to (pde).
- Using verification argument,

$$V(x,m) = \frac{x^{1-\gamma}}{1-\gamma}(u^*(m))^{-\gamma}, \quad (x,m) \in \mathbb{R}^2_+.$$

Main Result

Suppose $0 < \gamma < 1$ and $\bar{c} := \frac{\delta}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r > 0$. If

$$g\left((g')^{-1}\left(\frac{1-\gamma}{\gamma}\right)\right) < \beta, \tag{1}$$

then

$$V(x, m) = \frac{x^{1-\gamma}}{1-\gamma} u^*(m)^{-\gamma} \quad \text{for all } (x, m) \in \mathbb{R}_+^2,$$

where $u^* : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is the unique nonnegative, strictly increasing solution to

$$u(m)^2 - \left(\frac{\delta + (1 - \zeta^{1-\gamma})m}{\gamma} + \left(1 - \frac{1}{\gamma}\right) r\right) u(m)$$
$$+ m u'(m) \left(\sup_{h \geq 0} \left\{ g(h) - \frac{1-\gamma}{\gamma} \frac{u(m)}{m u'(m)} h \right\} - \beta\right) = 0.$$

Main Result (conti.)

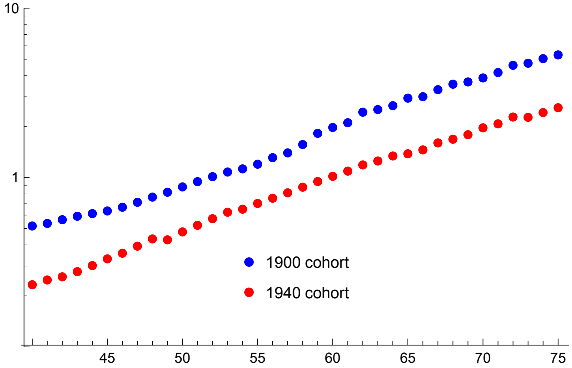
Furthermore, $(\hat{c}, \hat{h})$ defined by

$$\hat{c}_t := u^*(M_t), \quad \hat{h}_t := (g')^{-1}\left(\frac{1-\gamma}{\gamma}\frac{u^*(M_t)}{M_t(u^*)'(M_t)}\right), \quad t \geq 0,$$

are optimal strategies.

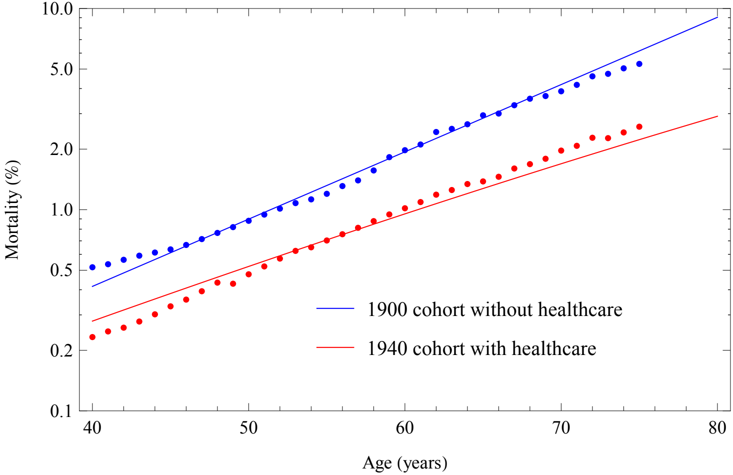
CALIBRATION

- Take *efficacy* function as $g(h) := a\frac{h^q}{q}$, with $a > 0$, $q \in (0, 1)$.
- Take $r = 1\%$, $\delta = 1\%$, $\gamma = 0.67$, $\zeta = 50\%$ from literature.
- Calibrate β, m_0, a, q to mortality rate data:



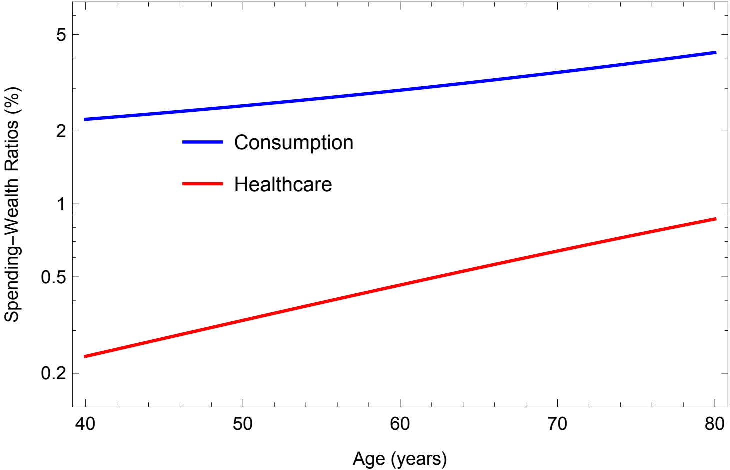
- $\implies \beta = 7.7\%,\ m_0 = 0.019\%,\ q = 0.46,\ a = 0.1.$

Longer Lives



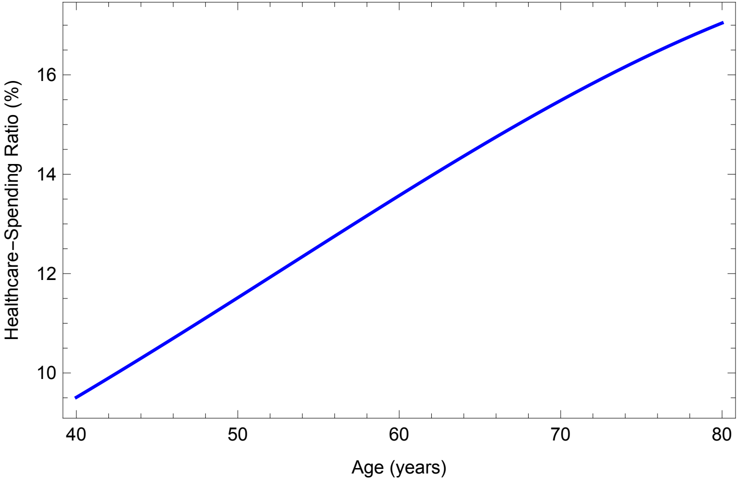
- Model explains decline in mortality at old ages.

Optimal Strategies



- Healthcare negligible in youth.
- Increases faster than consumption (in log scale!)

Healthcare as Fraction of Spending



- Convex, then concave; rises quickly to contain mortality.
- Slows down when cost-benefit declines.

THANK YOU!!

Q & A

Preprint available @ ssrn.com/abstract=2808362

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