15^{th} Annual SIAM Front Range Applied Mathematics Student Conference

March 2, 2019

Registration: 8:30 - 9:00

Morning Session I - Room 4017 9:00 - 11:35

9:00 - 9:20	Peter Rosenthal *University of Colorado, Boulder*	Breaking the Peierls-Nabarro Barrier with Topological Insulators
9:25 - 9:45	Adam Binswanger *University of Colorado, Boulder*	Oblique Dispersive Shock Waves in Steady Shallow Water Flows
9:50 - 10:10	Ryan Marizza & Jessica Harris *University of Colorado, Boulder*	Theory and Numerical Simulation of Interacting Linear Waves and Nonlinear Mean Flows in a Viscous Fluid Conduit
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Sophia Potoczak *Colorado State University*	An Operator Splitting Method for the Quantum Liouville-BGK Equation
10:50 - 11:10	Tristan Neighbors *University of Colorado, Colorado Springs*	The Lump Solitons of the KPI Equation
11:15 - 11:35	Yulong Li *University of Wyoming*	Investigation on Solutions of Fractional Diffusion-Advection-Reaction Equations with Variable Coefficients

Morning Session II - Room 4125 9:00 - 11:35

9:00 - 9:20	Daniel Bielich *University of Colorado, Denver*	Improving the Performance of Iterative Solvers
9:25 - 9:45	Kiera van der Sande *University of Colorado, Boulder*	Mesh Generation using Closest Point Method
9:50 - 10:10	Aryeni Tilsa *University of Wyoming*	Generalized Finite Element Method for Elliptic Two-Point BVP with an Interface
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Fortino Garcia *University of Colorado, Boulder*	Wave Equation Based Iterative Scheme for Helmholtz Problems

10:50 - 11:10	Mingyu Hu *University of Colorado, Boulder*	Taming the CFL number for Discontinuous Galerkin Methods by Local Exponentiation: Numerical Examples
11:15 - 11:35	Oleksii Beznosov *University of New Mexico*	Spin dynamics in modern electron storage rings: Mathematical and computational aspects

Morning Session III - Room 4113
9:00 - 11:35

9:00 - 9:20	Joseph Geisz *University of Colorado, Boulder*	Using Topology to Detect Bifurcations
9:25 - 9:45	Alexandria Ronco *University of Colorado, Denver*	Efficient estimation of ancestry proportions using genotype frequencies
9:50 - 10:10	Vladimir Vintu *University of Colorado, Boulder*	The Growth of Language Speakers in the next 50 years
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Gessner Soto	The Lotka Model: One Century Later
10:50 - 11:10	Emily Heavner *Colorado State University*	Introduction and Numerical Simulation of Dynamical System Lung Model
11:15 - 11:35	Liam Coulter *Colorado State University*	Synthetic Aperture Source Localization

Break: 11:35 - 11:45

Lunch: 11:45 - 12:25, Room 1600

Plenary Address: 12:30 - 1:30, Room 1600

Dr. Jan S. Hesthaven **How to Predict a Tsunami**

Chair of Computational Mathematics and Simulation Science,
Mathematics Institute of Computational Science and Engineering,
EPFL Switzerland

Group Photographs at 1:30

Afternoon Session I - Room 4017
1:45 - 3:20

1:45 - 2:05	Jordan R. Hall *University of Colorado, Denver*	CU Denver Student-Led Seminar
2:10 - 2:30	Michael Pilosov *University of Colorado, Denver*	Scientific Computation for Research and Teaching with Jupyter
2:35 - 2:55	Hanqing Li, Alice Wu, & Jack Gu *Colorado College*	Raising the Dragons: MCM Problem A
3:00 - 3:20	Sadie Graves & Christopher Padgett *Metropolitan State University*	MCM Problem A
3:25 - 3:45	Adrian Ward & Xavi Dominguez *Colorado College*	Winging it: A Tail of Dragon Energy Expenditure and Resource Use, MCM A

Afternoon Session II - Room 4125
1:45 - 3:20

1:45 - 2:05	Antony Pearson *University of Colorado, Boulder*	Latent Weights and Contamination in Symbolic Data
2:10 - 2:30	Francis Baffour-Awuah Junior *University of Vermont*	Contamination in Poissonian Datasets
2:35 - 2:55	Lewis R. Blake *Colorado School of Mines*	Scaling the Multi-Resolution Approximation (MRA) to Massive Spatial Data Sets
3:00 - 3:20	Anthony Sun *Colorado School of Mines*	Quantifying the role of heterogeneity in late-life mortality plateau and high early-life mortality

Afternoon Session III - Room 4113
1:45 - 2:55

1:45 - 2:05	Lucas Laird *University of Colorado, Boulder*	Resolvability of Hamming Graphs with Applications to Computational Biology
2:10 - 2:30	Richard C. Tillquist *University of Colorado, Boulder*	Multilateration of Random Networks with Community Structure
2:35 - 2:55	Wenjuan Zhang *University of Colorado, Denver*	Polynomial Chaos and Karhunen Loeve Expansion
3:00 - 3:20	Tyler Schuessler *University of Colorado, Boulder*	A Stochastic Prediction of the h-index

Plenary Speaker

12:30 - 1:30, 1600

How to Predict a Tsunami

Jan S. Hesthaven

Chair of Computational Mathematics and Simulation Science
MATHICSE, EPFL Switzerland

During the last decades, earthquake driven tsunamis have impacted the lives of millions and resulted in financial losses in the billions. Some of this devastation could be avoided if one could reliably predict the impact of tsunamis as an integral part of tsunami warning system, giving time to evacuate people and high value asserts as needed. In this talk we discuss the models and computational elements of a simulation tool to enable the prediction of tsunami arrival time on a global scale. The flexibility of the formulation allows for the use of a fully non-conforming discretization, opening the path to efficient adaptive computations. We illustrate the properties of the scheme through a series of simple one-dimensional tests before validating the method for the simulation of large-scale tsunami events on the rotating sphere by performing numerical simulations of several historical large scale events and compare our results to real-world data. By considering both static and dynamic earthquake models, we demonstrate that the method is able to predict arrival times and wave amplitudes accurately, even over long distances.

About the Speaker

Dr. Hesthaven currently holds the Chair of Computational Mathematics and Simulation Science within the Mathematics Institute of Computational Science and Engineering (MATHICSE) at École Polytechnique Fédérale de Lausanne (EPFL) in Switzerland. He was the Founding Director of the Center for Computation and Visualization (CCV) and Deputy Director of the Institute of Computational and Experimental Research in Mathematics (ICERM), at Brown University, the newest NSF Mathematical Sciences Research Institute. In March 2014 he was elected SIAM Fellow for contributions to high-order methods for partial differential equations. For more info, please visit: https://mcss.epfl.ch.

MORNING SESSION I

BREAKING THE PEIERLS-NABARRO BARRIER WITH TOPOLOGICAL INSULATORS

Peter Rosenthal
Advisors: Mark J. Ablowitz, Justin T. Cole, & Pi-pi Hu
University of Colorado, Boulder

Unidirectionally traveling edge waves can be found in helically varying honeycomb lattices, referred to as topological insulators. These topologically protected modes propagate around lattice boundary defects without backscatter (total transmission). In a deep lattice regime, the system approaches a discrete and nonlinear tight-binding approximation. One consequence of this is the appearance of the so-called Peierls-Nabarro (PN) barrier which is characterized by the slowing and eventual stopping of solitary waves. Hence the PN barrier and traveling edge waves seem to be incompatible. We intend to address the relationship between the PN barrier and topologically protected edge modes. It is found that the effect of topological protection on these modes can actually beat the PN barrier's effect; while the barrier wants to slow the solitary waves down, topological protection is stronger and keeps the wave's speed and direction consistent. Topological protection is not guaranteed however, and can be "turned off" in which case, for highly nonlinear modes, the PN barrier does appear to slow them down and eventually stop.

OBLIQUE DISPERSIVE SHOCK WAVES IN STEADY SHALLOW WATER FLOWS

Adam Binswanger
Advisors: Patrick Sprenger & Mark A. Hoefer
University of Colorado, Boulder

Steady shallow water flows are studied for a boundary value problem that corresponds to the deflection of a supercritical flow of a thin sheet of water past a slender wedge. Due to surface wave dispersion, the ensuing steady structure is a spatially extended, oscillatory pattern referred to as an oblique dispersive shock wave (DSW), which can be approximated as a modulated nonlinear wavetrain limiting to an oblique solitary wave at one edge and small amplitude harmonic waves at the other edge. This corner wedge boundary value problem is modeled by a weakly nonlinear model of KdV-type that incorporates higher order dispersion. Asymptotic analysis, numerical simulations, and an in-house shallow water experiment demonstrate evidence of a bifurcation in the flow pattern as a control parameter (the wedge angle) is varied. The Bond number, B, measuring the effects of surface tension relative to gravity, characterizes the bifurcation and is controlled by appropriate variation of water depth. The bifurcation, a result of higher order dispersion, occurs near $B = 1/3$, corresponding to a fluid depth of approximately 5 mm, and is a transition between classical and non-classical DSW profiles. They are differentiated by the monotonicity or lack thereof of the solitary wave edge as well as the structure of the modulated nonlinear wavetrain that ensues.

THEORY AND NUMERICAL SIMULATION OF INTERACTING LINEAR WAVES AND NONLINEAR MEAN FLOWS IN A VISCOUS FLUID CONDUIT

Ryan Marizza & Jessica Harris
Advisors: Michelle Maiden & Mark A. Hoefer
University of Colorado, Boulder

A theoretical and numerical analysis is described for the interactions of linear, small amplitude, dispersive waves with evolving, nonlinear mean flows, specifically: smooth expansion waves in a viscous fluid conduit. Analysis of such interactions has been developed for waves described by the Kortweg-de Vries (KdV) equation in the context of shallow water waves. In this poster, a similar analysis is applied to linear wave-mean flow interactions for the conduit equation that models a viscous fluid conduit—the cylindrical, free interface between two miscible, Stokes fluids with high viscosity contrast. A condition on the linear wave's wave-number pre and post interaction determines whether the linear wave will be transmitted through or be trapped by the mean flow. This analysis is complemented by direct numerical solutions of the conduit equation.

AN OPERATOR SPLITTING METHOD FOR THE QUANTUM LIOUVILLE-BGK EQUATION

Sophia Potoczak
Advisor: Olivier Pinaud
Colorado State University

I will introduce the quantum Liouville-BGK equation which models a statistical ensemble of electrons. An important characteristic of the quantum Liouville-BGK equation is the inclusion of the collision operator, which requires the definition of a local quantum equilibrium operator that is obtained by minimizing a free energy functional under a local electron density constraint. The numerical difficulties lie in the nonlinear collision term and are addressed by using a splitting scheme in which the nonlinearity becomes a perfectly linear term. The specifics of the splitting scheme will be presented, including convergence analysis and numerical results.

THE LUMP SOLITONS OF THE KPI EQUATION

Tristan Neighbors
Advisor: Sarbarish Chakravarty
University of Colorado, Colorado Springs

The Kadomstev-Petviashvili equation with positive dispersion (or KPI equation) models certain nonlinear wave phenomena ranging from water waves to nonlinear optics. Solutions of KPI also appear as potentials of the non-stationary Schrdinger equation, an important equation in quantum mechanics. Of particular interest are the lump soliton solutions of KPI, which are so named because they consist of distinct lumps which can interact nonlinearly with one another. The dynamics of these lumps has been the chief concern of our research. Thus far, we have studied the special cases of 2 and 3 interacting lumps. In each case, we were able to derive asymptotic formulae for the locations and heights of each lump as functions of time. The accuracy of these formulae was verified against the exact solutions. This research moves us closer to a more general theory describing the qualitative features of solutions involving any number of lumps. Of particular interest is a phenomenon wherein the lumps seem to align themselves along perpendicular lines in the plane for sufficiently large values of time.

INVESTIGATION ON SOLUTIONS OF FRACTIONAL DIFFUSION-ADVECTION-REACTION EQUATIONS WITH VARIABLE COEFFICIENTS

Yulong Li
Advisor: Victor Ginting
University of Wyoming

Fractional-order differential equations (FDEs) have been playing an increasingly important role in modeling many classes of physical phenomena and other practical applications. Compared to integer-order differential equations, FDEs seem to be more intricate to deal with due to the appearance of non-local operators. In this talk, I will focus on fractional diffusion advection reaction equation with variable coefficients and talk about the existence and uniqueness of solutions and its regularity. Furthermore I will point out the striking feature of the structure of solution which opens an instructive standpoint for the author to investigate other types of FDEs.

MORNING SESSION II

IMPROVING THE PERFORMANCE OF ITERATIVE SOLVERS

Daniel Bielich
Advisor: Julien Langou
University of Colorado, Denver

Orthogonalization schemes are critical for the convergence and the performance of many iterative solvers. (Typically Arnoldi-based Krylov methods.) In this talk we revisit the use of Householder reflections in the context of Krylov solvers. We provide efficient implementations in the context of one right-hand side but also in the context of block iterative methods. Our orthogonalization suite also has application in the context of dense linear algebra.

MESH GENERATION USING CLOSEST POINT METHOD

Kiera van der Sande[1]
Co-authors: Colin Macdonald[2]
[1] ***University of Colorado, Boulder***
[2] ***University of British Columbia***

Mathematical models, posed as partial differential equations (PDEs) on surfaces, arise in many areas in the natural and engineering sciences. In almost all but the simplest cases analytical solutions for these surface PDEs do not exist; efficient numerical methods are required. Many of these methods require a mesh. For efficiency and accuracy it is often necessary to automatically adapt the mesh to the surface geometry or the behaviour of the solution. We propose an application of the closest point method for solving PDEs on surfaces to the problem of adaptive mesh generation, which simplifies the surface problem and does not rely on any particular surface parameterization.

A mesh generator typically takes a physical domain Ω_p and transforms it to a computational domain Ω_c. Points $\boldsymbol{\xi}$ are generated in Ω_c and the inverse mapping back to Ω_p will form a mesh of points $\boldsymbol{x}$. Different mesh generators use different transformations.

We consider the Winslow variable diffusion mesh generator as a sample problem. The Winslow generator produces meshes that are generally smooth and seek to equidistribute mesh density over the domain. On a surface S in $\mathbb{R}^3$, this requires the solution of the elliptic PDEs

$$\begin{aligned} \nabla_S \cdot (\rho(\boldsymbol{x})\nabla_S \xi) = 0, \quad \text{for } \boldsymbol{x} \in S, \\ \nabla_S \cdot (\rho(\boldsymbol{x})\nabla_S \eta) = 0, \quad \text{for } \boldsymbol{x} \in S, \end{aligned} \tag{1}$$

where $\boldsymbol{\xi} = [\xi(\boldsymbol{x}), \eta(\boldsymbol{x})]$ is the coordinate mapping from $\Omega_p \subset S$ to $\Omega_c \subset S$, ∇_S is the surface gradient operator, and $\nabla_S \cdot$ is the surface divergence operator. The equations are subject to problem-specific boundary conditions which prescribe a boundary correspondence between Ω_p and Ω_c. $\rho(\boldsymbol{x})$ is a scalar mesh density function which characterizes where additional mesh resolution is needed.

The closest point method allows for intrinsic differential operators in a surface PDE to be replaced with their analogous Cartesian operators in the embedding space. Function values on the surface are mapped to Cartesian grid points in the embedding space. Solutions in the embedding space can be shown to solve the original problem when restricted back to the surface. Our numerical implementation uses straightforward finite difference methods on uniform computational grids enveloping the surface.

We demonstrate the effectiveness of the method through convergence on a curve in $\mathbb{R}^2$ and then obtain solutions on surfaces in $\mathbb{R}^3$ with various density functions $\rho(\boldsymbol{x})$. Non-homogenous Dirichlet boundary conditions are introduced to the closest point formulation to explore the effects of the surface boundary. Finally we consider more complex meshes using density functions that include a time dependent level set function and an anisotropic diffusion tensor.

This work illustrates that the closest point method can be used to perform basic meshing of open curved surfaces using Winslow's variable diffusion, hence introducing the algorithms and laying the groundwork for future work on more general surface mesh generators.

GENERALIZED FINITE ELEMENT METHOD FOR ELLIPTIC TWO-POINT BVP WITH AN INTERFACE

Aryeni Tilsa
Advisor: Victor Ginting
University of Wyoming

This study is inspired by a paper by Deng ,Q. and Calo, V. discussing about the generalized finite element methods (GFEMs) for the two-point boundary value problems with an interface. The idea is to develop robust and stable numerical methods for solving the problems that have the discontinuous diffusion coefficient at the interface. This is done by enlarging the standard FEM space with certain enrichment functions. Performance of the method is illustrated by several numerical examples. Some future work will also be discussed.

WAVE EQUATION BASED ITERATIVE SCHEME FOR HELMHOLTZ PROBLEMS

Fortino Garcia[1]
Advisor: Daniel Appelö[1]
Collaborators Olof Runborg[2] [1] ***University of Colorado, Boulder***
[2] ***Royal Institute of Technology in Stockholm***

Designing efficient iterative solvers for the Helmholtz equation is notoriously difficult and has been the subject of much research. The main two difficulties in solving the Helmholtz equation are (1) the resolution requirements and (2) the highly indefinite character of the discretized system of equations. We introduce a novel idea that enables the use of time domain methods for wave equations to design frequency domain Helmholtz type solvers. Our approach yields an underlying linear operator corresponding to a symmetric positive definite matrix allowing us to both solve a nice coercive problem (rather than a highly indefinite Helmholtz problem) and acceleration of convergence via methods such as conjugate gradient (CG). Numerical examples with various discretization techniques are presented.

TAMING THE CFL NUMBER FOR DISCONTINUOUS GALERKIN METHODS BY LOCAL EXPONENTIATION: NUMERICAL EXAMPLES

Mingyu Hu
Advisor: Daniel Appelö
University of Colorado, Boulder

High-order discontinuous Galerkin (DG) methods are spectrally accurate and thus are especially beneficial for solving wave propagation problems. However, a drawback with DG methods is that the Courant-Friedrichs-Lewy (CFL) number decreases rapidly with increasing basis polynomial order. In this talk, the speaker will propose a local low-rank approximation to the exponential integrator for time-stepping to tame the CFL condition for stiff hyperbolic partial differential equations (PDEs). The construction of the novel time-stepper is motivated by the nature of wave propagation. The accuracy of this time stepping method is inherited from the exponential integrator and the local property of it allows lower costs and parallel implementation. The method is therefore expected to be useful in design and inverse problem where many solves of the PDE are required. Potential applications include (1) design of meta-material from unit cell response to the propagation of electromagnetic waves, (2) inner-cavity geometry update, and (3) extension to implementation of helmholtz solvers by exact controllability method. In this talk, we demonstrate the stability and error convergence of the method in 1D Maxwell?s equation on uniform grids. Moreover, the speaker will demonstrate the use of the fast forward-solver in full-waveform inversion (FWI). Results of source inversion for 1D Maxwell?s equation will be presented.

SPIN DYNAMICS IN MODERN ELECTRON STORAGE RINGS: MATHEMATICAL AND COMPUTATIONAL ASPECTS

Oleksii Beznosov[1]
Advisors: Daniel Appelö[2], Desmond Barber[3], James Ellison[1], Klaus Heinemann[1]
[1] ***University of New Mexico***
[2] ***University of Colorado, Boulder***
[3] ***Deutsches Elektronen-Synchrotron***

Particle accelerators have applications in science, industry and medicine. The largest particle accelerators serve the purpose of testing the standard model of particle physics and extensions of this model as well. Typically, these large accelerators carry beams of electrons or protons (and their antiparticles, positrons and anti-protons) and have led to the discovery of many elementary particles, e.g. quarks, gluons and the Higgs particle. Since electrons and protons carry a spin vector, some accelerators are designed to align the spin vectors of a beam in order to get a net spin polarization of the beam. A spin polarized beam allows for additional types of experiments to be conducted in an accelerator. Our focus is on circular electron accelerators in particular the proposed 30-mile CEPC in China and planned extension FCC-ee of the 17 mile LHC at CERN (the Higgs particle was discovered at the LHC in 2012). The design and performance of spin-polarized circular accelerator is not an easy task. The standard theoretical approach to study the polarization is based on the so-called DK formulas introduced in 1973. However, this approach may not be valid at the high energies of future machines like CEPC and FCC-ee.

Our project is aimed at the polarization of electron beams in high energy storage rings. We plan to check the validity of the DK formulas and extend them if needed by numerically integrating the full Bloch equation. The full Bloch equation is a system of partial differential equations which is more fundamental than the DK formulas and is believed to contain all the relevant physics. The full Bloch equations can be classified as an advection-diffusion problem in 3 degrees of freedom i.e. in 6 phase space variables + time variable. Our idea is first to apply the method of averaging to the orbit and spin evolution stochastic differential equations which, in turn, allows the efficient numerical simulation of the averaged problem. We use the fast Fourier transform for the angular dimensions and introduce the parallel in Fourier modes numerical algorithm. The results of numerical simulation for a 1 degree of freedom model circular accelerator will be presented.

MORNING SESSION III

USING TOPOLOGY TO DETECT BIFURCATIONS

Joseph Geisz
Advisors: Elizabeth Bradley, James Meiss, & Nicole Sanderson
University of Colorado, Boulder

Topological Data Analysis (TDA) in general studies the “shape of a data set. Does the data have "holes? Does it have distinct "pieces? These are difficult questions to answer considering data sets are discrete collections of 0-dimensional points. Persistent Homology allows us to interpret this data topologically. A simplicial complex is constructed from the data dependent on a single parameter which in effect represents the scale at which the data is observed. Noting how the topology changes with the parameter reveals many characteristics of the data set. Studying data in this way has provided new strategies for classifying data, analyzing time series, and generally gaining insight into the structure of large datasets despite the computational difficulties involved. The abstraction of these concepts allows for these techniques to be applied in a variety of contexts. In this presentation I will give an introduction to Persistent Homology and some of the parameters involved in its computation. Then, in order to illustrate the concepts, I will introduce synthetic data sets created from a known chaotic dynamical system. Finally I will share the results of experiments run with the Persistent Homology on Embedded Time Series (PHETS) python library and show how this method of analysis can give insight into bifurcation and change points of data sets.

EFFICIENT ESTIMATION OF ANCESTRY PROPORTIONS USING GENOTYPE FREQUENCIES

Alexandria Ronco, James Vance, Ian Arriaga Mackenzie, Greg Matesi
Advisor: Audrey E. Hendricks ***University of Colorado, Denver***

Public genetic data enables efficient and more equitable access, transforming genetic and medical re- search. Due to privacy concerns, data is often provided by group genotype frequency rather than individ- ually. Grouping can mask important information, such as fine-scale ancestry, and imprecise ancestry in- formation may lead to misdiagnoses and incorrect genetic associations. We present a method to estimate hidden ancestry proportions in genotype frequency data. With more ancestries and therefore dimensions in the data, estimating these proportions quickly and precisely is problematic. Our method employs Se- quential Quadratic Programming, which we utilize in Pythons Sequential Least Squares Quadratic Pro- gramming (SLSQP) package, an iterative minimization algorithm for constrained, nonlinear problems. Grid search took > 1 hour to produce estimates for 4 ancestries at a 1% precision; SLSQP gives results in seconds at $< 0.1\%$ precision. We apply our method to open databases which provide precise ancestry in- formation.

THE GROWTH OF LANGUAGE SPEAKERS IN THE NEXT 50 YEARS

Vladimir Vintu
Advisor: Beth Malmskog
Collaborators: David Cui & Xinling Dai
Colorado College

When does someone become a language speaker? Or, better put, when does one consider themselves a speaker of a language? Is it after 100 hours of learning? Is it because of a few words and phrases learned from a friend? Whether or not someone speaks a language can be subjective and can depend on the cultural settings and individual perspectives. This is why, in our opinion, it is of high importance to structurally understand the factors that might contribute to any changes to the number of speakers of a given language over time.

In this presentation, we aim to find a model that predicts the ten most spoken languages in 50 years from now, and we consider both native speakers and non-native speakers. The target of our presentation is to sensibly combine population growth rates, migration behaviors and education policies around the world, and reason how these three factors have an essential role in the fluctuations of the number of speakers over time. Because of the constantly increasing migration rates all over the world, and the recent phenomenon of globalization, the migration behavior will stand as a dominant factor for the changes that we wish to find.

We divided our research into four categories. In the first category are the so-called Domestic Native Speakers, which are the native speakers in 50 years from now that would not have migrated to a country with another native language. We used the Malthusian growth model to determine the growth rate of these people, and registered our results. A further demographic analysis was made in order to adjust the growth model to the current worldwide situation. In the second category are the New Native Speakers, which are the native speakers in 50 years from now that had become native speakers through migration (of their families, prior to their birth). The paper explains how these two terms will accurately predict the total number of native speakers 50 years from now.

The third category is our Education Sector, in which we focus on the likelihood that students around the world will learn any of these languages. In this section, we investigate various policies around the world, and predict the number of learners in the next 50 years. By gathering the data from each sector, we conclude with the lists that we initially aimed for.

THE LOTKA MODEL: ONE CENTURY LATER

Gessner Soto

One century later, the prevalence of computational machinery allows a substantial sub-set of our species to have access to hardware that is able to adequately handle the numerous sequential steps required to analyze the solution structure of related differential-equation relationships. An arrangement related by Dr. Ilya Prigogine in the third edition of "An Introduction to Thermodynamics of Irreversible Processes" - one whose structure is simply an ornamentative modification to the arrangement Dr. Alfred Lotka formally related in his 1,920 contribution "Undamped Oscillations Derived from the Law of Mass Action" - was un-packed more fully with the aid of contemporary machinery. Two interests currently seem to both have had been nourished and continue to be nourished with this project: 1) exposure to and development of numerical algorithms related to the deconstruction of the solution sets associated with collections of differential-equation relationships (a two dimensional differential-equation collection with three parameters was partially deconstructed) and 2) an explicit grounding in the terms and logic utilized in the non-equilibrium thermodynamics lineage (one of the aspects Dr. Prigogine emphasized in the analysis he relates is the associated thermodynamically grounded conjectures and their place within the context of the associated kinetic differential-equation system). The generality potential associated with these thermodynamic constructs merits the investment.

INTRODUCTION AND NUMERICAL SIMULATION OF DYNAMICAL SYSTEM LUNG MODEL

Emily Heavner
Advisor: Jennifer Mueller
Colorado State University

We introduce a dynamical system modeling airflow in the lungs based on Chellaboina et al., International Journal of Control, Vol 83, 2010, with numerical data to simulate the volume of a lung during inhalation and exhalation. We investigate the role of several physical parameters including airway resistance, pressure, and number of branches in the alveolar tree. The accuracy and disadvantages of this lung model as well as potential improvements are discussed. In a future study, we will compare this lung model and electrical impedance tomography, a medical imagining technique, with data collected to improve manual jet ventilation, a respiratory assistance tool.

SYNTHETIC APERTURE SOURCE LOCALIZATION

Liam Coulter
Advisor: Margaret Cheney
Colorado State University

In recent years the subject of localizing sources of electromagnetic radiation has become a rich research topic in the mathematics and signal processing communities. In this talk I will present a method for localizing point sources in a target scene using a technique from Synthetic Aperture Radar (SAR) image processing. Specifically, the method makes use of Time Difference of Arrival (TDOA) data from two or more receivers, at least one of which is in motion, and a filtered backprojection operator, to reconstruct an image of the target scene.

AFTERNOON SESSION I

CU DENVER STUDENT-LED SEMINAR

Jordan R. Hall
University of Colorado, Denver

In Fall 2016, graduate students in the CU Denver Math and Stats department began an experimental seminar by and for students. Nearly three years later, students know Student-Led Seminar (SLS) as a bi-weekly staple in our department, providing a safe, judgement-free zone for communicating mathematics. SLS has provided a venue for undergraduate and graduate math students to practice high-stakes presentations, showcase the results of passion projects, and teach other graduate students about pedagogy and web design. In this talk, we share the results of our nearly-three-year SLS programming effort, and make an argument for implementing similar student-only time and space in student life.

SCIENTIFIC COMPUTATION FOR RESEARCH AND TEACHING WITH JUPYTER

Michael Pilosov
University of Colorado, Denver

We will discuss the Jupyter ecosystem of web-applications that can dramatically lower the programming barrier-to-entry for students and new scientists alike. The infrastructure that has recently been deployed in the CU Denver Department of Mathematics & Statistics will be used as a case-study to demonstrate the capabilities and benefits of using modern open-source software in educational and research settings.

RAISING THE DRAGONS: MCM PROBLEM A

Hanqing Li, Alice Wu, & Jack Gu
Colorado College

Dragon is a magical and mysterious creature in the epic fantasy novel A Song of Ice and Fire. If those dragons live in real world, how to raise them based on local environment and ensure their compatibility with climate conditions would be great concerns in a scientific perspective. In this paper, we build a logistic growth model with energy exchange in order to look into this problem and design an appropriate way to raise the dragons.

First, we build a logistic growth model that analyzes dragon characteristics, habits, and diet. In order to estimate the rate of growth, several equations are used to reveal the relationship between different parameters. The parameters, including the range of activity area, the length of dragon, energy required to survive per year and body mass, are assumed and referred to different animal models. This basic model shows that overall, the weight of dragon grows exponentially in the first 15 years, then gradually slows down and converge to its growth limit.

Second, in order to simulate the dragons interaction with their environment, we improve our previous model by incorporating energy cost for one dragon to maintain its body temperature into the model. To estimate energy cost, we calculate the energy loss through heat convection and radiation, which is correlated to the following parameters: environment temperature, body temperature, surface area of the dragon body, humidity, wind speed, air viscosity, thermal conductivity of air and intensity of solar radiation. The improved model (logistic growth model with energy exchange) demonstrates a longer growth period for the dragon to reach a lower growth limit compared with the basic model, which indicates that energy loss do significantly impact the dragons growth rate.

Third, we apply our model to predict the growth of dragon under different climate condition, considering environment temperature and solar radiation as two of the main factors. According to our analysis, the dragon will grow slower and smaller in arctic region and will grow faster and larger in arid region. In addition, we analyze how the emergence of the dragons can impact the ecological system from the perspective of fire and species.

Finally, we select parameters: amount of food available to the dragon, size of activity area available and body temperature for sensitivity analysis.

MCM PROBLEM A

Sadie Graves & Christopher Padgett
Metropolitan State University of Denver

If dragons lived in today's world, ecological obstacles would surface, pressuring the sustainability of Earth's fragile ecosystem. Introducing a predator at the top of the food chain could have detrimental effects to the natural world and its biological relationships. The dragons would have to be careful about where they hunt and which animals they target as to not effect population stability of a particular species. If dragons flew around hunting whomever and whatever they like, extinction rates would likely spike worldwide, endangering the natural cycle of life. Harvesting food for the dragons, like cattle and sheep, is an ideal solution to prevent an impact on ecosystems, however, this is unrealistic since dragons enjoy hunting and are trainable only to some degree. Thus, migration is necessary to spread out the destruction these dragons can cause to the surrounding environments and limit negative population impact towards any one species. Modeling a migration pattern that enables sustainability for the dragons as well as the ecosystems at play is the most realistic approach. In order to model an appropriate migration path, it is crucial to consider the daily caloric intake required for a full-size dragon to execute tasks such as flying, breathing fire and resting. This estimation enables an understanding of how many calories a dragon might consume on a daily, monthly and yearly basis. Although the growth of the dragons is a point of interest, the model created is representative of a migration path necessary once the dragons are fully grown. Based on the estimated size the full-grown dragons, an appropriately sized area of the globe where these dragons can migrate from season to season throughout the year can be approximated. The area cannot be too large because the caloric cost of flying and migrating is high. The area cannot be too small, or we will see unwanted changes to surrounding animal populations. To minimize migration distance, three "dragon bases" were designed, one in an arctic region, one in an arid region and the last in a temperate or tropical region. These bases can sustain three dragons for a certain amount of time during a certain time of year. Based on population cycles of deer, cattle and buffalo herds it is known where food is abundant and the dragons can hunt without worry, provided they stick to their strict daily diet. The bases act as mini communities focused on annual harvesting of sheep, cattle and bison depending on their region. They will also act as a home-base for the dragons as they travel a certain radius from the central community. The analysis will provide a thorough breakdown of the thought process and assumptions necessary to conclude the amount of annual caloric resources necessary to sustain a dragon.

WINGING IT: A TAIL OF DRAGON ENERGY EXPENDITURE AND RESOURCE USE, MCM PROBLEM A

Adrian Ward, Xavi Dominguez, & Ruyi Yang
Advisor: Jane McDougall
Colorado College

This year we completed question A of the MCM. The question was mainly asking to model the energy expenditure of three dragons from Game of Thrones. We created a growth function based on data from reptiles and dinosaurs, then applied that to an energy expenditure function taking in sheep for energy and using it for fire breathing, metabolism and flying. In the future, we would have distributed our time differently in order to create more models, like a predator prey model and a Bernoulli distribution model to take into account the chance of wild fire.

AFTERNOON SESSION II

LATENT WEIGHTS AND CONTAMINATION IN SYMBOLIC DATA

Antony Pearson
Advisor: Manuel E. Lladser
University of Colorado, Boulder

Notions of "contamination" in samples from continuous probability distributions have been well-studied, with work dating back to the 19th century and culminating famously with Huber's notion of robust statistics. These traditional models of contamination typically view data as a mixture model

$$P = (1 - \epsilon) \cdot P^* + \epsilon \cdot R,$$

where P^* is an idealized model (typically Gaussian), and ϵ is a small probability of sampling from a contaminating distribution R. With continuous data there is often phenomenological and theoretical justification for assuming contamination is Gaussian.

In contrast, when dealing with symbolic data such as high-throughput genomic sequences, there is little reason to assume a particular model of contamination. We describe a new notion which we call the "latent weight" of a probabilistic source P with respect to some family $\mathcal{Q}$ of well-structured probability models. In short, a latent weight of P can describe how often data from P appear to be distributed according to some desired model, e.g. a Poisson distribution, or in the case of multivariate data, an independent or exchangeable distribution. We provide theoretical results on latent weights and demonstrate the practicality of the concept by examining exchangeability in DNA methylation datasets.

CONTAMINATION IN POISSONIAN DATASETS

Francis Baffour-Awuah Junior[1]
Collaborator: Antony Pearson [2]
Advisor: Manuel E. Lladser [2]
[1] ***University of Vermont***
[2] ***University of Colorado, Boulder***

In 1898, Ladislaus Bortkiewicz noted that the aggregate occurrence of events with low frequency in a large population appears to follow a Poisson distribution, even when the probabilities of the events differ. Because of this, the Poisson distribution is a natural though not perfect model for many real-world experiments taking values in the sample space

$\{0, 1, 2, \ldots\}$. In this presentation, we show how to estimate the largest expected proportion of data from an arbitrary discrete probability distribution that can be attributed to a single Poissonian model. We do so by identifying and computing the largest weight that can be attributed to a Poisson component in a mixture model, and allowing the "contaminated" component to have an arbitrary distribution. We develop a methodology that allows efficient computation of the contaminating proportion and the underlying Poisson rate and explore the estimation of these quantities when the source in question is observed only indirectly through data. This work is in collaboration with Antony Pearson and supervised by Manuel Lladser.

SCALING THE MULTI-RESOLUTION APPROXIMATION (MRA) TO MASSIVE SPATIAL DATA SETS

Lewis R. Blake
Advisor: Dorit Hammerling
Colorado School of Mines

Building from previous work, we implement a parallel version of the Multi-Resolution Approximation (MRA) using a shallow-tree approach designed for distributed computing envi- ronments and HPC systems. The formal derivation of the MRA and efficient implementa- tions are important advancements in spatial statistics for a number of reasons. With increasing amounts of data being produced (e.g., by remote sensors and numerical models), the statistical and computational techniques to handle these data sizes historically have lagged. The MRA has been shown to be one of the most computationally efficient and accurate methods to analyze large spatial data sets. By performing statistical inference in a distributed fashion across nodes, we aim to significantly increase the amount of data which can be utilized for analysis. In our novel shallow-tree parallelization scheme, the user specifies the first few levels to be computed in serial. After serial computations, workers are assigned specific sections of the data with which to perform parallel calculations. By using codistributed arrays in our paral- lelization scheme, we reduce the amount of memory and communication overhead by ensuring that calculations are statistically independent. We apply our shallow-tree MRA to data sets on the order of 8 million observations with the objective of scaling our model to data sets on the order of one-hundred million observations.

QUANTIFYING THE ROLE OF HETEROGENEITY IN LATE-LIFE MORTALITY PLATEAU AND HIGH EARLY-LIFE MORTALITY

Anthony Sun[1] & Eric Sun[2]
[1] ***Colorado School of Mines***
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High early life mortality (HELM) and Late-life mortality plateau (LLMP) are significant deviations from Gompertzian mortality observed in many biological populations. However, the correspondence between these population-level deviations and the intrinsic mortality trajectories in biological aging has yet to be determined. An attractive explanation is that mortality heterogeneity in Gompertzian population may result in these population-level deviations. Here we show that both HELM and LLMP can be derived from genetic and environmental heterogeneity. Using computational models of a heterogeneous Gompertzian population, we show that LLMP and HELM can arise from heterogeneity in Gompertzian parameters and is sensitive to the mean value of these parameters. For a diverse set of organisms (D. melanogaster, C. elegans, H. sapiens, H. glaber), we determine the contribution of heterogeneity to LLMP and HELM. We provide the first method for quantifying the contribution of heterogeneity to species-specific deviations from Gompertzian mortality.

AFTERNOON SESSION III

RESOLVABILITY OF HAMMING GRAPHS WITH APPLICATIONS TO COMPUTATIONAL BIOLOGY

Lucas Laird
Advisors: Manuel E. Lladser & Richard C. Tillquist
University of Colorado, Boulder

This work studies the resolving sets and metric dimension of Hamming graphs. A set of vertices in a graph is called resolving if every other vertex in the graph can be uniquely identified by its distances to the set. The metric dimension is the smallest of such resolving sets. It is known that metric dimension is NP-hard on general connected simple graphs

but on some families such as trees, fully connected graphs, and paths, it is much easier to compute. Hamming graphs are one such family of connected simple graphs and it is unknown if computing their metric dimension is NP-hard or not. In this work we show that a set is resolving on a Hamming graph if and only if there is only the trivial solution to a constrained linear system. Further, we represent these constraints as polynomial equations and provide an algorithm for showing a set is resolving using Gröbner bases.

MULTILATERATION OF RANDOM NETWORKS WITH COMMUNITY STRUCTURE

Richard C. Tillquist
Advisor: Manuel E. Lladser
University of Colorado, Boulder

In a network endowed with the geodesic distance between nodes, the minimal number of nodes needed to multilaterate the network (i.e., to uniquely identify all nodes based on distances to the selected nodes) is called its *metric dimension*. While this quantity has been studied for many kinds of graphs, its behavior on the Stochastic Block Model (SBM) ensemble has not. Graphs in this ensemble typically exhibit simple community structure and, as a result, are of interest in a variety of areas. Here we derive probabilistic bounds for the metric dimension of random graphs generated by the SBM and describe an algorithm to find—with high probability—subsets of nodes capable of multilaterating these graphs. Our methods are tested on SBM ensembles with parameters extracted from several real-world networks including the Karate Club and Political Blogs networks.

This research was partially funded by NSF IIS grant 1836914. The authors acknowledge the BioFrontiers Computing Core at the University of Colorado–Boulder for providing High-Performance Computing resources (funded by National Institutes of Health 1S10OD012300), supported by BioFrontiers IT group.

POLYNOMIAL CHAOS AND KARHUNEN LOEVE EXPANSION

Wenjuan Zhang
Advisor: Troy Butler
University of Colorado, Denver

Part I: I will use several examples to show how Polynomial Chaos works to solve ordinary differential equations. Besides, I am going to compare the results from Polynomial Chaos and Monte Carlo method when solving the same problem. Part II: I will present some examples using Karhunen Loeve expansion and talk about my interest in connecting it to inverse problems.

A STOCHASTIC PREDICTION of the h-index

Tyler Schuessler
Advisor: Manuel Lladser
University of Colorado, Boulder

The h-index, or Hirsch index, is a bibliometric index intended to measure the impact of a researcher's scholarly output. It is defined as the largest integer k such that an author has k papers with k or more citations each. In this talk, we characterize the asymptotic probability distribution of the h-index for a researcher with m papers. The model is informed by a random citation network of n papers following Price's preferential attachment model. We demonstrate the correspondence of our prediction on both simulated data and real citation networks from the e-prints repository arXiv.org.