14^{th} Annual SIAM Front Range Applied Mathematics Student Conference

March 3, 2018

Registration: 8:30 - 9:00

Morning Session I - Room 4017
9:00 - 11:35

9:00 - 9:20	Michael Pilosov *University of Colorado, Denver*	That's an Inverse Problem!
9:25 - 9:45	Jordan Hall *University of Colorado, Denver*	Comparing Two Dimension Reduction Techniques
9:50 - 10:10	Lucas Ortiz *University of Colorado, Denver*	BlockTrace-Modernizing the ATF Gun Trace Program with Blockchain
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Jeremy L. Thompson *University of Colorado, Boulder*	On Performance and Portability for Generic Finite Element Interfaces
10:50 - 11:10	Zhuoran Wang *Colorado State University*	New Finite Element Method for Darcy Flow on General Polygonal Meshes
11:15 - 11:35	Graham Harper *Colorado State University*	New Finite Element Methods for Elasticity on Quadrilateral and Hexahedral Meshes

Morning Session II - Room 4125
9:00 - 11:35

9:00 - 9:20	Lauren M. Nelsen *University of Denver*	Many Edge-Disjoint Rainbow Spanning Trees in General Graphs
9:25 - 9:45	Eric Culver *University of Colorado, Denver*	tP_3 Saturated Graphs with a Small Number of Cycles
9:50 - 10:10	Charles Viss *University of Colorado, Denver*	Circuit Walks in Integral Polyhedra
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Zhishen Huang *University of Colorado, Boulder*	Escaping Saddle Points and the Application on Dictionary Learning Problem

10:50 - 11:10	Stephan Patterson *University of Colorado, Denver*	Divide-and-Conquer Algorithm for Discrete Barycenters
11:15 - 11:35	Stetson Zirkelbach *University of Colorado, Denver*	Build Order Optimization in Star Craft 2 Using the Critical Path Method

Morning Session III - Room 4113
9:00 - 11:35

9:00 - 9:20	Ian Klasky *University of Colorado, Boulder*	Quantifying Resilience in One-Dimensional Dynamical Systems
9:25 - 9:45	Emileigh L. Willems *University of Colorado, Denver*	Genome-wide Association of Metabolic Syndrome in a Multi-ethnic Study
9:50 - 10:10	Anthony Pearson *University of Colorado, Boulder*	Exchangeability of Non-Exchangeable Datasets
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Ian Char *University of Colorado, Boulder*	Analysis of Minimal Automata for Generalized Strings
10:50 - 11:10	Richard C. Tillquist *University of Colorado, Boulder*	Low-Dimensional Embeddings of Hamming Graphs for Biological Sequence Data
11:15 - 11:35	Jonathan W. Lavington *University of Colorado, Boulder*	A Zipper Model for CRISPR-Cas9 Genetic Engineering Systems

Break: 11:35 - 11:45

Lunch: 11:45 - 12:25, Room 2500

Plenary Address: 12:30 - 1:30, Room 2600

Dr. Benedetto Piccoli — **Modeling of Crowd Dynamics**

Department of Mathematical Sciences,
Rutgers University-Camden

Group Photographs at 1:30

Afternoon Session I - Room 4017
1:45 - 3:20

1:45 - 2:05	Adam Binswanger & Maxwell Lambek *University of Colorado, Boulder*	Stationary Oblique Dispersive Shock Waves in Shallow Water
2:10 - 2:30	Alyssa Ortiz *University of Colorado, Colorado Springs*	Soliton Solutions of Certain Reductions of the Matrix Nonlinear Schrodinger Equation
2:35 - 2:55	Evan Shapiro *University of Colorado, Denver*	Predicting the Scaling Relationship Between Ion Heat Conduction and Plasma Reactor Size Under Uncertainty Using Surrogate Models
3:00 - 3:20	Harry Dudley *University of Colorado, Boulder*	Sensitivity and Bifurcation Analysis of a Differential-Algebraic Equation Model for a Microbial Electrolysis Cell

Afternoon Session II - Room 4125
1:45 - 3:20

1:45 - 2:05	Angel Farguell Caus *University of Colorado, Denver*	Interpolation of Fire Perimeters by a Fire Spread Model
2:10 - 2:30	J. Matthew Maierhofer *University of Colorado, Boulder*	Lifetime-Limited Memory: A Novel Approach to Event Sequence Prediction
2:35 - 2:55	Nikhil Krishnan *University of Colorado, Boulder*	A Jump-Diffusion Model for Optimal Foraging
3:00 - 3:20	Vladimir Vintu *Colorado College*	The Growth of Language Speakers in the Next 50 Years

Afternoon Session III - Room 4113
1:45 - 2:55

1:45 - 2:05	Liam Healy, John Mckinstry, & Joseph Leavitt *Metropolitan State University of Denver*	Mathematical Contest in Modeling (MCM) Problem C - Energy Profile
2:10 - 2:30	Christina Ebben *University of Colorado, Denver*	Linear Programming Methods in Radiation Therapy Treatment Planning
2:35 - 2:55	Vincent C. Herr *University of Colorado, Denver*	Quarter-Square Multiplication in Early Microprocessors

Plenary Speaker

12:30 - 1:30, 2600

Modeling of Crowd Dynamics

Dr. Benedetto Piccoli

Department of Mathematical Sciences, Rutgers University - Camden

During the holiday shopping season, malls seem to be as crowded as busy city streets. It's a pedestrian traffic jam from store to store as people try to navigate the pathways that will lead them to the perfect holiday gift, and maybe even a bargain. Trying to get around or through droves of people isn't just a science perfected by savvy shoppers. The modeling of such phenomena need to take into account various aspects, for instance psychology, which studies the cognitive processes behind the action of walking; and mathematics, which attempts to quantify the laws that govern the way crowds of people move or interact. Topics will include:

1. The phenomenology of crowd dynamics: self-organization, patterns and cognitive processes.
2. Experiments with crowds: what to measure, how to measure, experimental settings.
3. Modeling crowd dynamics: choice of the scale, ODE and PDE models, new measure theory approach.
4. Measure-theory multi-scale models: math behind, properties of the model, numerics and simulations.

About the Speaker

Dr. Benedetto Piccoli is the Joseph and Loretta Lopez Chair of Mathematics at Rutgers University - Camden. Dr. Piccoli is the author of several books and scholarly articles on crowd dynamics and traffic flow. Piccoli's recent book, Multiscale Modeling of Pedestrian Dynamics (Springer, 2014), brings together two disciplines when analyzing crowd dynamics: psychology, which studies the cognitive processes behind the action of walking; and mathematics, which attempts to quantify the laws that govern the way crowds of people move or interact. For more information, visit http://piccoli.camden.rutgers.edu/

MORNING SESSION I

THAT'S AN INVERSE PROBLEM!

Michael Pilosov
Advisor: Troy Butler
University of Colorado, Denver

We demonstrate the pervasiveness of inverse problems in the scientific disciplines. To illustrate the breadth of possible applications, we will rely solely on the use of visual illustrations and animations. By seeing the connections between mathematics and other fields, the audience will learn how to recognize inverse problems "in the real world."

COMPARING TWO DIMENSION REDUCTION TECHNIQUES

Jordan Hall
Advisor: Varis Carey ***University of Colorado, Denver***

When working with high dimension models, it is often useful to create a surrogate model of lower dimension. With a surrogate model in hand, we may be able to compute Quantities of Interest more efficiently. Two dimension reduction techniques – Principal Component Analysis (PCA) and Active Subspace analysis – will each be used to reduce the dimension of a high-dimensional, algebraic model. At their core, PCA and Active Subspace analysis can be roughly described as eigenvalue analysis of a covariance matrix. The example presented is intended highlight the subtle differences, along with the pro's and con's of each approach.

BLOCKTRACE-MODERNIZING THE ATF GUN TRACE PROGRAM WITH BLOCKCHAIN

Lucas Ortiz
University of Colorado, Denver

Only 65% of requests submitted to the ATF's gun trace center are able to linked to the original point of sale. This is a direct result of the byzantine and outdated system of decentralized paper and microfilm records that was established due to a lack of trust between the federal government, gun retailers, and gun purchasers. I propose a modernization of the gun trace system based on blockchain technology which can maintain privacy and allow quick and reliable traces while simultaneously solving the problem of a lack of trust.

ON PERFORMANCE AND PORTABILITY FOR GENERIC FINITE ELEMENT INTERFACES

Jeremy L. Thompson[1]
Advisor: Jed Brown[1]
Collaborators: Jean-Sylvain Camier[2], Tzanio Kolev[2], Veselin Dobrev[2], & Thilina Rathnayake[3]
[1] ***University of Colorado, Boulder***
[2] ***Lawrence Livermore National Laboratory***
[3] ***University of Illinois, Urbana-Champaign***

One of the challenges with high-order finite element and spectral element methods is that a global sparse matrix is no longer a good representation of a high-order linear operator, both with respect to the FLOPs needed for evaluation and the memory transfer needed for a matrix-vector multiply. Thus, high-order methods require a new operator description that still represents a linear (or more generally nonlinear) operator. libCEED is an extensible library that provides a portable algebraic interface and optimized implementations suitable for high-order operators. libCEED's operator description is easy to incorporate in a wide variety of applications, without significant refactoring of the discretization infrastructure. We introduce the libCEED API and discuss some preliminary results in comparing performance to native implementations in production software, such as Nek5000 and MFEM.

NEW FINITE ELEMENT METHOD FOR DARCY FLOW ON GENERAL POLYGONAL MESHES

Zhuoran Wang
Advisors: James Liu & Simon Tavener
Colorado State University

This talk presents a novel weak Galerkin (WG) finite element method for Darcy flow on general convex polygonal meshes. In this method, constants are used to approximate pressure in element interiors and on edges. The discrete weak gradients of these constant basis functions are defined in H(div)-subspaces that are constructed from the normalized and Wachspress coordinates. These discrete weak gradients are then used to approximate classical gradients. This new WG method has nice features, e.g., local mass conservation and continuous normal fluxes. It results in symmetric positive-definite discrete linear systems. First order convergence in pressure, velocity, and flux will be demonstrated by numerical experiments.

NEW FINITE ELEMENT METHODS FOR ELASTICITY ON QUADRILATERAL AND HEXAHEDRAL MESHES

Graham Harper
Advisors: James Liu & Simon Tavener
Colorado State University

We investigate lowest order weak Galerkin (WG) finite element methods (FEMs) for solving the linear elasticity equation on quadrilateral and hexahedral meshes. This approach allows for a space of piecewise constants to be considered, for which the notion of a discrete weak gradient is introduced. This is accomplished through integration by parts. These discrete weak gradients are defined for the local Raviart-Thomas space $RT_{[0]}^{d}(d = 2, 3)$. Discrete weak divergence and strain are defined similarly. Finite element schemes are developed for both the grad-div and the strain-div formulations, and theoretical analysis is presented for the grad-div case. Theoretical and numerical results both demonstrate first order convergence in displacement, stress, and dilation (divergence of displacement) when the solution has full regularity and the mesh is asymptotically parallelogram or parallelopiped. Methods for reducing the global linear system are also discussed.

MORNING SESSION II

MANY EDGE-DISJOINT RAINBOW SPANNING TREES IN GENERAL GRAPHS

Lauren M. Nelsen
Co-author: Paul Horn
University of Denver

A rainbow spanning tree in an edge-colored graph is a spanning tree in which each edge is a different color. Carraher, Hartke, and Horn showed that for n and C large enough, if G is an edge-colored copy of K_n in which each color class has size at most $n/2$, then G has at least $\lfloor n/(C \log n) \rfloor$ edge-disjoint rainbow spanning trees. Here we strengthen this result by showing that if G is *any* edge-colored graph with n vertices in which each color appears on at most $\delta \cdot \lambda_1/2$ edges, where $\delta \geq C \log n$ for n and C sufficiently large and λ_1 is the second-smallest eigenvalue of the normalized Laplacian matrix of G, then G contains at least $\left\lfloor \frac{\delta \cdot \lambda_1}{C \log n} \right\rfloor$ edge-disjoint rainbow spanning trees.

tP_3 SATURATED GRAPHS WITH A SMALL NUMBER OF CYCLES

Eric Culver
Co-authors: David Brown, Julie Sánchez, & Brent Thomas
University of Colorado, Denver

Graphs are tP_3 saturated when the graph does not contain a tP_3, but adding any edge creates a tP_3 (where we use tP_3 to refer to t disjoint copies of the path on 3 vertices). We investigate the structure of tP_3 saturated graphs, showing a number of results, culminating in a complete list of tP_3 saturated graphs with at most 4 cycles.

CIRCUIT WALKS IN INTEGRAL POLYHEDRA

Charles Viss
Advisor: Steffen Borgwardt
University of Colorado, Denver

As a generalization of the edge directions of a polyhedron, circuits play a fundamental role in the theory of optimization due to their connection to combinatorial algorithms and the efficiency of the Simplex method. In this talk, we cover a brief introduction to the fields of combinatorial and circuit diameter. Then, to gain a better understanding ofcircuits, we introduce a hierarchy for integral polyhedra based on behaviors exhibited by their circuit walks. Many problems in combinatorial optimization fall into the most interesting categories of this hierarchy—steps of circuit walks in the associated polyhedra only stop at integer points, at vertices, or follow actual edges. We classify several classical families of integral polyhedra within the hierarchy, including 0/1-polytopes, polyhedra defined by totally unimodular matrices, and more specifically matroid polytyopes, transportation polytopes, and partition polytopes. Finally, we prove several equivalent characterizations of the non-degenerate polytopes in the bottom level of the hierarchy where all circuit walks are edge walks.

ESCAPING SADDLE POINTS AND THE APPLICATION ON DICTIONARY LEARNING PROBLEM

Zhishen Huang

Advisor: Stephen Becker
University of Colorado, Boulder

Machine learning problems such as neural network learning, matrix factorization and tensor decomposition require local minimization of a non-convex problem. This local minimization is challenged by the presence of saddle points of the objective functions, of which there can be many and from which classical descent methods may take large number of iterations to escape. Two classes of methods have been proposed for escaping from saddle points. One class, the gradient-based methods, adds noise into the first-order information. The other class, the Hessian-based, exploits second-order information to update iteration.

The first part of this talk will review two classes of saddle-point-escaping methods by discussing algorithm, the proof of their convergence property and their efficiency.

The second part of this talk will discuss the target of generalising the existing saddle-point-escaping methods to non-smooth, non-convex objective functions, and the application of saddle-point-escaping methods on the dictionary learning problem, which assumes the target function $\min_{\boldsymbol{D},\boldsymbol{x}} \frac{1}{2}\|\boldsymbol{Y} - \boldsymbol{D}\boldsymbol{X}\|_F^2 + \mu\|\boldsymbol{X}\|_1 + \gamma\|\boldsymbol{D}\|_F$.

DIVIDE-AND-CONQUER ALGORITHM FOR DISCRETE BARYCENTERS

Stephan Patterson
Co-author: Steffen Borgwardt
University of Colorado, Denver

Discrete barycenters are the optimal solutions to mass transport problems for discrete measures. They arise in applications of operations research and statistics. These barycenters can be computed by linear programming. However, the programs scale exponentially in the number of measures, and so it is of interest to trade a small approximation error for a large reduction of computational effort. We study an heuristic for the computation of approximate barycenters that will maintain two notable, favorable properties of exact barycenters: a sparse support and a non-mass splitting optimal transport.

This divide-and-conquer approach splits the computation of a barycenter into several parts. Exact barycenters are computed for subsets of the measures, which then are merged using another barycenter computation. We exhibit a significant advantage in computation times and favorable practical results. The approach gives an exact optimum in dimension 1, and a single merging step provides a 3-approximation, independent of the dimension.

BUILD ORDER OPTIMIZATION IN STAR CRAFT 2 USING THE CRITICAL PATH METHOD

Stetson Zirkelbach
Co-author: Malik Odeh
University of Colorado, Denver

The critical path method is a useful tool in linear programming for scheduling and project modeling by finding the longest path of planned activities. Real time strategy video games rely on a build order to grow an economy support their side and ultimately win the game against their opponent. In this talk we examine how the resource, unit and tech tree requirements of the various playable factions can be expressed in a linear model to solve the critical path problem for a given build order.

MORNING SESSION III

QUANTIFYING RESILIENCE IN ONE-DIMENSIONAL DYNAMICAL SYSTEMS

Ian Klasky[1]
Advisor: Mary Lou Zeeman[2]
Collaborators: Torey Lee[2], Alana Hoyer-Leitzel[3], & Kate Meyers[4]
[1] ***University of Colorado, Boulder***
[2] ***Bowdoin College***
[3] ***Mount Holyoke College***
[4] ***University of Minnesota***

In common discourse, resilience is often taken to be the ability of a system to undergo disturbance(s) while retaining useful characteristics. A mathematically precise definition of resilience would allow us to assess and compare quantities of resilience in a range of natural and engineered systems. In this study, we develop a method for quantifying the resilience of single population systems to regular disturbance based on its growth dynamics.

GENOME-WIDE ASSOCIATION OF METABOLIC SYNDROME IN A MULTI-ETHNIC STUDY

Emileigh L. Willems[1]
Co-authors: Jia Y. Wan[2], Karen L. Edwards[2], & Stephanie A. Santorico[1,3]
[1] ***University of Colorado, Denver***
[2] ***University of California Irvine***
[3] ***Human Medical Genetics and Genomics Program***

Metabolic Syndrome (MetS) is defined as a clustering of metabolic risk factors, which when present together, can increase the risk of cardiovascular disease, Type 2 Diabetes, and stroke. Examples of these metabolic risk factors include: abdominal obesity, high triglyceride (TG) levels, high fasting glucose levels, low levels of HDL cholesterol, and high blood pressure. Samples from the GENetics of NonInsulin-dependent Diabetes mellitus (GENNID) Study serve as a characterization of Type 2 Diabetes multiplex families across four diverse ethnic groups: European-American (# families = 79 , # individuals = 519), Japanese-American (# families = 17, # individuals = 132), Mexican-American (# families = 113, # individuals = 610), and African-American (# families = 73, # individuals = 288). Since the families within the study were acquired due to the large prevalence of Type 2 Diabetes, MetS occurs with high frequency within the GENNID Study. The goal of our study is to identify associations between common variants in the genome and eight quantitative MetS traits (body weight, waist circumference, systolic and diastolic blood pressure, TG, HDL, fasting glucose, and fasting insulin) in the GENNID Study. The analysis procedure used to detect genome-wide associations within the four GENNID samples will be discussed, along with association results from the European-American and African-American groups. Comparisons will be made for the results of the two ethnic groups, and the need for trans-ethnic meta-analysis methods will be illustrated.

EXCHANGEABILITY OF NON-EXCHANGEABLE DATASETS

Anthony Pearson
Advisor: Manuel E. Lladser
University of Colorado, Boulder

Exchangeability of a random vector, the invariance of its probability distribution to "shuffling" the variables in the vector, is a common assumption in a variety of statistical contexts. For instance, null hypotheses in a permutation test implicitly assume exchangeability of the data. Recent research in machine learning has addressed special cases of testing the null hypothesis that a stream of data is exchangeable. Here we outline a totally different view, where any discrete probability distribution P, exchangeable or not, may be written as a mixture containing an exchangeable component E with a certain weight $0 \leq \lambda \leq 1$. In particular, the larger the λ, the more exchangeable the data from P. We describe the largest weight an exchangeable component of P can have, and give results on statistical estimation when a random sample from it is available. As proof of concept, we analyze synthetic and real data.This material is based upon work supported by the National Science Foundation Graduate Research Fellowship under Grant No. (2016198773) and partially funded by the NSF IGERT 1144807 grant.

ANALYSIS OF MINIMAL AUTOMATA FOR GENERALIZED STRINGS

Ian Char
Advisor: Manuel E. Lladser
University of Colorado, Boulder

In the world of bioinformatics, a significant problem is to recognize certain patterns, or *biological motifs*, in a DNA sequence. A standard approach for finding such motifs is to employ a graph-like structure called a *deterministic finite automaton* (DFA). The practicality of this approach, however, relies on the DFA having manageable size. To address this problem, we focus on a class of biological motifs known as *generalized strings*. Given such a string, we present a new algorithm that constructs its associated minimal DFA. Since this algorithm constructs the DFA directly, it facilitates the analysis of the resulting automaton. Indeed, we present results on what can be said about the size of the minimal DFA when the generalized string is drawn at random. This project has been partially funded by EXTREEMS-QED NSF grant 1407340.

LOW-DIMENSIONAL EMBEDDINGS OF HAMMING GRAPHS FOR BIOLOGICAL SEQUENCE DATA

Richard C. Tillquist
Advisor: Manuel E. Lladser
University of Colorado, Boulder

Many machine learning algorithms require that examples be embedded in a Euclidean space. When faced with symbolic datasets, the choice of embedding is important as naive mapping functions have the potential to bias results. In addition, conventional embedding approaches for many types of symbolic data, particularly biological sequence data, may be unnecessarily high dimensional. Here we address these issues by exploiting the graph-theoretic notion of "metric dimension" as the basis for embedding functions. Given a graph $G = (V, E)$, a set $R \subseteq V$ is called *resolving* if, $\forall u, v \in V$, $\exists r \in R$ such that $d(u, r) \neq d(v, r)$ where $d(\cdot, \cdot)$ is the shortest path distance between two nodes in G. The metric dimension of G, denoted $\beta(G)$, is the size of minimal resolving sets on G. The problem of determining the metric dimension of a general graph is NP-complete. However, efficient approximation algorithms and bounds associated with specific families of graphs do exist. Motivated by the problem of characterizing targets of the Dengue virus protease (i.e. an enzyme which breaks down proteins), a task which has proven difficult, we develop a concise representation of octapeptides, sequences of eight amino acids. This representation is based on a novel bound on the metric dimension of Hamming graphs and provides significant dimensionality reduction in comparison to common biological sequence embeddings while uniquely identifying all octapeptides.

This work was partially funded by the NSF IGERT 1144807 grant. The authors acknowledge the BioFrontiers Computing Core at the University of Colorado Boulder for providing High Performance Computing resources (NIH 1S10OD012300) supported by BioFrontiers IT.

A ZIPPER MODEL FOR CRISPR-Cas9 GENETIC ENGINEERING SYSTEMS

Jonathan W. Lavington
Advisor: Manuel E. Lladser
University of Colorado, Boulder

CRISPR-Cas9 is a genetic engineering technique pioneered in the early 2000s that researchers use to make targeted changes to an organism's DNA. This technique has created new avenues for the manipulation of microbial metabolisms (e.g. for ethanol production), and the possibility of effective gene therapy. Unfortunately, while great strides have been made over the last decade, this method is still prone to inaccuracies, often generating sub-optimal gene editing. One of the primary reasons this method has become so popular in recent years however, is due to its simplicity. Given a target site, the enzyme Cas9, along with a small guide RNA, attacks and cleaves small portions of the host genome. The efficiency with which it cleaves a specific sequence depends on the number and location of mismatches between the guide sequence and the desired target. In this talk, we present a differential equations model combined with a discrete Markov chain model to predict the efficiency of CRISPR-Cas9 based on various factors. Using global optimization approaches, we then fit experimental results by finding optimal parameters for our bio-probabilistic model. Using these parameters we then predict off-target effects within a new experimental data set, and use these predictions to design the guide RNA within new experiments. This work is done in collaboration with K. Tarasava at the Gill Lab (ChBE) at CU-Boulder, and has been partially funded by the EXTREEMS-QED NSF grant 1407340.

AFTERNOON SESSION I

STATIONARY OBLIQUE DISPERSIVE SHOCK WAVES IN SHALLOW WATER

Adam Binswanger & Maxwell Lambek
Advisors: Patrick Sprenger and Mark Hoefer
University of Colorado, Boulder

Supersonic flow past a slender wedge is a canonical problem of interest in compressible gas dynamics. We consider a similar problem with supercritical flow of water past a wedge, where the medium is largely dissipationless and dispersive. In this system, the interaction results in an oscillatory, steady pattern understood to be an oblique dispersive shock wave (DSW). These patterns are similar to phenomena observed in nonlinear fiber optics, superfluids, and Bose-Einstein condensates. One engineering application of our theory is to reduce the damage on a spillway by minimizing the shock waves caused by wedge-like objects.

We develop a scalar model equation with multiple scale analysis of the steady, two dimensional generalized Serre equations. The system models fully nonlinear water waves with weak dispersion. The multiple scales analysis results in a steady Korteweg-de Vries (KdV) equation where the boundary conditions at the edge can be formulated as a step initial condition. This mapping of a boundary value problem to an initial value problem is possible because of the wave-like or hyperbolic property of the dispersionless equations. From the KdV equation, we can find relationships between parameters of this physical system, such as the relation between upstream and downstream depth, leading and trailing angles of the oblique DSW, and the range of wedge angles in which the model is valid. In-house experiments used to test the validity of this model using a table-top water table experiment will also be discussed.

SOLITON SOLUTIONS OF CERTAIN REDUCTIONS OF THE MATRIX NONLINEAR SCHRODINGER EQUATION

Alyssa Ortiz[1]
Advisor: Barbara Prinari[1]
Collaborators: Cornelis van der Mee[2] **& Marek Grabowski**[1]
[1] ***University of Colorado, Colorado Springs***
[2] ***Universita di Caglia***

We will apply the inverse scattering transform method to two novel reductions of the matrix nonlinear Schrodinger equation which are integrable, and which are the analog of the modified Manakov system with mixed signs of the nonlinear coefficients, i.e., a nonlinearity in the norm which is of Minkowski type, instead of Euclidean type. We will present various soliton solutions for these systems. We will classify one soliton solutions, discuss regularity conditions and investigate special solutions including double pole solutions, bound states, and two soliton solutions.

PREDICTING THE SCALING RELATIONSHIP BETWEEN ION HEAT CONDUCTION AND PLASMA REACTOR SIZE UNDER UNCERTAINTY USING SURROGATE MODELS

Evan Shapiro
University of Colorado, Denver

Global gyrokinetic models have been shown to accurately predict the ion conduction, χ_i, by incorporating microturbulence into the models. Extreme-scale, fixed-flux supercomputing simu- lations are beginning to simulate modes of operation relevant to next-generation (ITER,DEMO) reactors. Using a surrogate model to reduce the computational expense of XGC simulations, I conduct a predictive scan in ρ^{-1}, to ascertain whether or not the ion diffusivity χ_i scales in a Bohm or gyro-Bohm fashion, and analyze the sensitivity of χ_i to perturbation in the heating model.

SENSITIVITY AND BIFURCATION ANALYSIS OF A DIFFERENTIAL-ALGEBRAIC EQUATION MODEL FOR A MICROBIAL ELECTROLYSIS CELL

Harry Dudley
Advisor: David M. Bortz
University of Colorado, Boulder

Microbial electrolysis cells (MECs) are a promising new technology for producing hydrogen cheaply, efficiently, and sustainably. The technology is based on microbial fuel cells in which bacteria oxidize an organic substrate to generate current, providing decreased electricity costs when compared to direct electrolysis. MECs are also more efficient than fermentation methods and can be fed fermentation effluent or cheap and readily available wastewater. However, to scale up this technology, we need a better understanding of the processes in the devices. In this effort, we present an index-one differential-algebraic equation (DAE) model of a microbial electrolysis cell with an algebraic constraint on current. We then perform sensitivity and bifurcation analysis for this DAE system. The sensitivity analysis yields temporal regions wherein reactor adjustments will have the largest impact on productivity. For the bifurcation analysis, we present results concerning the classification of transcritical bifurcations in the input flow rate. Overall, our conclusions provide guidance on optimizing performance of batch-fed and continuous-flow reactors. This work was partially supported by NSF grant DMS-1225878.

AFTERNOON SESSION II

INTERPOLATION OF FIRE PERIMETERS BY A FIRE SPREAD MODEL

Angel Farguell Caus
Advisor: Jan Mandel
University of Colorado, Denver

Assimilation of data into a fire-spread model is formulated as an optimization problem. The level set equation, which relates the fire arrival time and the rate of spread, is allowed to be satisfied only approximately, and we minimize a norm of the residual. Previous methods based on modification of the fire arrival time either used an additive correction to the fire arrival time, or made a position correction.

Unlike additive fire arrival time corrections, the new method respects the dependence of the fire rate of spread on diurnal changes of fuel moisture and on weather changes, and, unlike position corrections, it respects the dependence of the fire spread on fuels and terrain as well. The method is used to interpolate the fire arrival time between two perimeters by imposing the fire arrival time at the perimeters as constraints.

LIFETIME-LIMITED MEMORY: A NOVEL APPROACH TO EVENT SEQUENCE PREDICTION

J. Matthew Maierhofer
Advisor: Michael Mozer
University of Colorado, Boulder

In the modern digital environment, many data sources can be characterized as event sequences. These event sequences describe a series of events and an associated time of occurrence. Examples of event sequences include: the call log from a cell phone, an online purchase history, or a trace of musical selections. The influx of data has led many researchers to develop deep architectures that are able to discover event sequence patterns and predict future sequences (e.g., CT-GRU, Hawkes process memories), but none have shown a benefit over the state-of-the-art in the field, Long Short-Term Memory (Hochreiter and Schmidhuber, 1995), in prediction or classification tasks. LSTM is a recurrent neural network, a model that has an internal state which is updated as each new input is presented. This class of architecture has been shown to be particularly well suited for use with sequenced data, as it allows for the development of context based input. The LSTM was developed to observe and remember patterns in the data, but also retain the ability to forget previous patterns that become irrelevant to the task as the sequence continued. As a result, a forget gate was added to the network. This allowed it to learn to forget the irrelevant information, but introduced further issues due to memory leakage and flattening gradients restricting training. LSTM was developed to handle ordinal sequences, but can also be provided with time stamp as additional input to allow it the ability to handle event sequences as well. We propose a Lifetime-Limited Memory (LLM) architecture that operates under the notion that all information within a sequence is relevant for only a finite time period. The age, then, is used to determine how much of the memory should be retained via a hierarchy of leaky integrators with log linear spaced time constants. As the network trains, each cell linearly mixes the information from the different timescales, and determines the most relevant time scales for each event. We believe that this architecture will be better equipped to handle this specific class of tasks then more traditional methods because it incorporates temporal dynamics into its neuron activation functions and permits the storage and utilization of information at multiple time scales.

A JUMP-DIFFUSION MODEL FOR OPTIMAL FORAGING

Nikhil Krishnan
Advisor: Zachary Kilpatrick
University of Colorado, Boulder

We consider the movement of a forager on a one dimensional periodic lattice, where each location contains one unit of food. As the forager lands on sites with food, the forager consumes the food, leaving the sites empty. If the forager lands consecutively on S empty sites, then it will starve. The forager has two modes of movement. It can either diffuse, by moving with equal probability to adjacent points on the lattice, or it can jump uniformly randomly among the points on the lattice. We show that the lifetime T of the forager under either of these paradigms can be approximated by a linear function of the cover time when the starvation time S of the forager far exceeds the number of locations on the lattice. We also consider a hybridized approach, where the forager has a probability of either jumping or diffusing, and we show that the lifetime of the forager varies non-monotonically according to p_d, the probability of diffusion.

THE GROWTH OF LANGUAGE SPEAKERS IN THE NEXT 50 YEARS

Vladimir Vintu
Advisor: Beth Malmskog
Collaborators: David Cui, Xinling Dai
Colorado College

The data for the total number of people that speak a certain language might be easily misleading, due to the difficulty to accurately track down the exact number of speakers of each language. This is why, in our opinion, it is of high importance to structurally understand the factors that might contribute to any changes to the speakers of a given language over time.

In this presentation, we aim to introduce a model that would help us predict two lists: one with the top 10 most spoken languages in 50 years from now, and the other with the top 10 most spoken languages in 50 years from now in terms of native speakers. The target of our presentation is to sensibly combine population growth rates, migration behaviors and education policies around the world, and reason how these three factors occupy an essential role in the fluctuations of the number of given language speakers over time. Due to the constantly higher increases in migration all over the world, and the phenomena of globalization that the world has been recently facing, the migration behavior will stand as a dominant factor for the changes that we want to find.

We divided our research into four categories. In the first category fall the so-called Domestic Native Speakers, which are the native speakers in 50 years from now, directly descendants from current Native speakers. We used the Malthusian growth model and algebraic manipulations to determine the growth rate of these people, and registered our results. In the second category fall the New Native Speakers, which are the native speakers in 50 years from now that became native speakers due to recent migration to a different country, prior to their birth. The paper explains how these two terms will accurately predict the total number of native speakers 50 years from now.

The third category is our Education Sector, in which we focus on the likelihood that students around the world will learn any of the languages that we are interested in. In this section, we investigate various policies around the world, and predict the number of learners in the next 50 years. Last, but not least, the fourth category represents the variation in the number of Refugees, and how they might influence the total number of speakers of certain languages. For this section, we argued how the refugees will eventually undergo a logistic growth, which will further lead to a statistical failure.

By gathering all the data from each sector, we conclude with the initial lists that we aimed to reach.

AFTERNOON SESSION III

MATHEMATICAL CONTEST IN MODELING (MCM) PROBLEM C - ENERGY PROFILE

Liam Healy, John Mckinstry, & Joseph Leavitt
Metropolitan State University of Denver

As part of the Mathematical Modeling Contest, we had to review the energy consumption of the states within the South West Interstate Energy Compact and determine which would best use the new renewable energy. With the use of several technologies (e.g. SAS, Python, and the World Wide Web), we were able to build a firm understanding of how the consumption over these states evolved through 1960 and 2009. In addition, we were able to apply a cutting edge machine learning algorithm that helped us forecast the consumption of each state over the next 40 years. Come learn what it's like putting in 12 hour days for 5 straight days on a single problem with only lame math jokes to keep us sane.

LINEAR PROGRAMMING METHODS IN RADIATION THERAPY TREATMENT PLANNING

Christina Ebben
Advisor: Steffen Borgwardt
University of Colorado, Denver

In 2017, the American Cancer Society estimated nearly 1.7 million new instances of cancer in the United States. Linear programming methods for radiation therapy treatment planning is a fascinating and practical application of optimization research. And, given the changing technologies in the field of radiation oncology, the linear programming methods for cancer treatment continue to grow and the methods can be extended to other medical fields. We will discuss the background of the application including the procedure, the treatment area and how to classify human tissue in mathematical terms. Formulating an initial linear program and well defining the components of the model is essential to understanding the foundations of all linear programming methods in radiation therapy treatment planning. Once a deeper understanding of the baseline model is established we will explore some variations of the model including the dual linear program. Special attention is paid to obtaining communicable and meaningful results that can be implemented in cancer treatment planning. Understanding the pros and cons of each linear model, as well as the difficult task of interpreting the dual program, gives the methods discussed many areas to improve and be revised in future research.

QUARTER-SQUARE MULTIPLICATION IN EARLY MICROPROCESSORS

Vincent C. Herr
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8-bit microprocessors in the 1970's and 1980's had no built-in multiply command and ran at relatively slow clock speeds. One response to these limitations was quarter-square multiplication:
int((a+b)ˆ2/4)- int((a-b)ˆ2/4)=ab. We describe implementation of this algorithm, illustrate an example at the level of machine language commands, and compare it with shift-and-add binary integer multiplication. We recognize the quarter-square multiplication as the polarization identity in R, and offer a geometric interpretation.