13^{th} Annual SIAM Front Range Applied Mathematics Student Conference

March 4, 2017

Registration: 8:30 - 9:00

Morning Session I - Room 4017 9:00 - 11:35

9:00 - 9:20	James Folberth *University of Colorado, Boulder*	URV Factorization with Random Orthogonal System Mixing
9:25 - 9:45	Derek Driggs *University of Colorado, Boulder*	Tensor Tools for Machine Learning
9:50 - 10:10	Zhuoran Wang *Colorado State University*	Solving the Darcy equation by the lowest order weak Galerkin finite elements on quadrilatral and hybrid meshes
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Nathan Graber *University of Colorado, Denver*	Plane Graphs with Maximum Degree 7 and no Adjacent Triangles are Entirely 10-Colorable
10:50 - 11:10	Cat Myrant *University of Colorado, Denver*	Coloring Around Faces to Count Daisies
11:15 - 11:35	Michael Phillips *University of Colorado, Denver*	On the 123-Conjecture for Products

Morning Session II - Room 4125 9:00 - 11:35

9:00 - 9:20	Alex Masarie *Colorado State University*	A PDE Approach to Pattern Detection in Wildland Fire Resource Allocation Data
9:25 - 9:45	James Haley *University of Colorado, Denver*	Estimating the ignition point of wildfires
9:50 - 10:10	Devon Sigler *University of Colorado, Denver*	Smart Electric Vehicle Charging Under Uncertainty
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Justin Garrish *University of Colorado, Colorado Springs*	Mathematical Modeling of the HPA Axis

10:50 - 11:10	Ian Char *University of Colorado, Boulder*	Construction of Minimal Automata for Generalized Languages à la Aho-Corasick
11:15 - 11:35	Jonathan W. Lavington *University of Colorado, Boulder*	A Zipper Model for CRISPR-Cas9 Binding Systems

Morning Session III - Room 4113
9:00 - 11:35

9:00 - 9:20	Shannon Grubb *Colorado School of Mines*	Active Subspaces in Energy Modeling
9:25 - 9:45	Andrew Glaws *Colorado School of Mines*	Inverse Regression for Ridge Recovery
9:50 - 10:10	Jonathan Helland *Colorado School of Mines*	Exploring Active Subspaces in Neural Network Cost Functions
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Ellen Considine, Suyog Soti, & Emily Webb *University of Colorado, Boulder*	ICM PROBLEM F: Migration to Mars: Utopian Workforce of the 2100 Urban Society
10:50 - 11:10	Zach Grey *Colorado School of Mines*	Multi-objective shape optimization of transonic airfoils using active subspaces
11:15 - 11:35	Aaron Nielsen, Haoyue Zhang, & Lilong Wang *University of Colorado, Denver*	Analyzing FCQ Results Using Advanced Data Analytics

Break: 11:35 - 11:45

Lunch: 11:45 - 12:25, Room 2500

Plenary Address: 12:30 - 1:30, Room 2600

Dr. Paul Constantine
Department of Applied Mathematics and Statistics, Colorado School of Mines

Active Subspaces: Emerging Ideas for Dimension Reduction in Computational Science and Engineering Models

Group Photographs at 1:30

Afternoon Session I - Room 4017
1:45 - 3:20

1:45 - 2:05	Tommy McDowell *University of Colorado, Colorado Springs*	Numerical Study of Stem in (3142) KP Line Soliton Solution
2:10 - 2:30	Patrick Sprenger *University of Colorado, Boulder*	Non-classical dispersive shock waves in shallow water
2:35 - 2:55	Maximilian Ruth *University of Colorado, Boulder*	Perimeter Dynamics of a Two-Dimensional, Circular Soliton in a Magnetic System
3:00 - 3:20	Graham Harper *Colorado State University*	The Lowest Order Weak Galerkin Solver for the Darcy Equation on Hexahedral Meshes

Afternoon Session II - Room 4125
1:45 - 3:20

1:45 - 2:05	Jaden Pieper *University of Colorado, Boulder*	A Very Brief Introduction to Quantum Computing
2:10 - 2:30	Michael J. Schmidt *Colorado School of Mines*	A Kernel-based Lagrangian Method for Imperfectly-mixed Chemical Reactions
2:35 - 2:55	Lauren M. Nelsen *University of Denver*	Many edge-disjoint rainbow spanning trees in general graphs
3:00 - 3:20	Luke L. Nelsen *University of Colorado, Denver*	Erdős-Szekeres Online

Afternoon Session III - Room 4113
1:45 - 3:45

1:45 - 2:05	Christina Ebben & Keegan Schmitt *University of Colorado, Denver*	Educational Applications of Undergraduate Research Through Compiling a Database and Special Topics in Optimization Research
2:10 - 2:30	Izabel Aguiar, Jessica Deters, & Jacquie Feuerborn *Colorado School of Mines*	The Mathematics of Gossip

Plenary Speaker

12:30 - 1:30, 2600

Active Subspaces: Emerging Ideas for Dimension Reduction in Computational Science and Engineering Models

Dr. Paul Constantine

Department of Applied Mathematics and Statistics, Colorado School of Mines

Scientists and engineers use computer simulations to study relationships between a physical model's input parameters and its output predictions. However, thorough parameter studies—e.g., constructing response surfaces, optimizing, or averaging—are challenging, if not impossible, when the simulation is expensive and the model has several inputs. To enable parameter studies in these cases, the engineer may attempt to reduce the dimension of the model's input parameter space. Active subspaces are part of an emerging set of subspace-based dimension reduction tools that identify important directions in the input parameter space. I will (i) describe computational methods for discovering a model's active subspaces, (ii) propose strategies for exploiting the reduced dimension to enable otherwise infeasible parameter studies, and (iii) review results from several science and engineering applications. For more information, visit activesubspaces.org

About the Speaker

Paul G. Constantine is the Ben L. Fryrear Assistant Professor of Applied Mathematics and Statistics at Colorado School of Mines. He received his Ph.D. from Stanford University's Institute for Computational and Mathematical Engineering and spent two years as the John von Neumann Fellow at the Sandia National Laboratories' Computer Science Research Institute. His research interests include uncertainty quantification and dimension reduction for large-scale computer simulations. For more information, visit www.mines.edu/∼pconstan

MORNING SESSION I

URV FACTORIZATION WITH RANDOM ORTHOGONAL SYSTEM MIXING

James Folberth[1]
Advisor: Stephen Becker[1]
Collaborator: Laura Grigori[2]
[1]***University of Colorado, Boulder***
[2]***INRIA of Paris***

The unpivoted and pivoted Householder QR factorizations are ubiquitous in numerical linear algebra. A difficulty with pivoted Householder QR is the communication bottleneck introduced by pivoting. In this paper we propose using random orthogonal systems to quickly mix together the columns of a matrix before computing an *unpivoted* QR factorization. This method computes a URV factorization which forgoes expensive pivoted QR steps in exchange for mixing in advance, followed by a cheaper, unpivoted QR factorization. The mixing step typically reduces the variability of the column norms, and in certain experiments, allows us to compute an accurate factorization where a plain, unpivoted QR performs poorly. We experiment with linear least-squares, rank-revealing factorizations, and the QLP approximation, and conclude that our randomized URV factorization behaves comparably to a similar, known randomized rank-revealing URV factorization, but at a fraction of the computational cost. Our experiments provide evidence that our proposed factorization might be rank-revealing with high probability.

TENSOR TOOLS FOR MACHINE LEARNING

Derek Driggs
Advisor: Stephen Becker
University of Colorado, Boulder

Many data scientists are discussing the increasing size of modern data sets, but few are discussing their increasing complexity. MRI scans, healthcare records, and genome sequences are all examples of multi-dimensional data sets that today's data scientists would like to analyze. Unfortunately, most tools of data analysis are matrix-based, enforcing a two-dimensional structure on the data even when it is not natural to do so. In this talk, we will discuss how tensors can be used to extend existing algorithms into a multi-dimensional setting. In particular, we will present a new approach to a tensor-based robust principal component analysis with recovery guarantees.

SOLVING THE DARCY EQUATION BY THE LOWEST ORDER WEAK GALERKIN FINITE ELEMENTS ON QUADRILATERAL AND HYBRID MESHES

Zhuoran Wang
Advisors: Dr. James Liu & Dr. Simon Tavener
Colorado State University

This talk presents the lowest-order weak Galerkin (WG) finite element method for solving the Darcy equation on quadrilateral and hybrid meshes. In this approach, we approximate the pressure by constants inside elements and on edges. The discrete weak gradients of these constant basis functions are defined in local Raviart-Thomas spaces $RT_{[0]}$ for quadrilaterals or in RT_0 for triangles. The classical gradient will be approximated by these discrete weak gradients. This new WG finite element method is locally mass-conservative and produces continuous normal fluxes. The method is easy to implement and results in symmetric positive-definite discrete linear systems. Numerical experiments will be presented to demonstrate expected convergence in both full and low regularity cases.

PLANE GRAPHS WITH MAXIMUM DEGREE 7 AND NO ADJACENT TRIANGLES ARE ENTIRELY 10-COLORABLE

Nathan Graber
Collaborators: Axel Brandt, Michael Ferrara, Stephen Hartke, & Sarah Loeb
University of Colorado, Denver

Let G be a plane graph with maximum degree Δ. If every element in the vertex set of G can be colored with k colors so that any two adjacent vertices recieve different colors, then G is said to be k-colorable. Famously, every planar graph is 4-colorable. An extension of this coloring idea for plane graphs is entire coloring. If every element in the sets of vertices, edges, and faces of G can be colored with k colors so that any two adjacent or incident elements have distinct colors, then G is said to be entirely k-colorable. In 2011, Wang and Zhu asked if every simple, plane graph $G \neq K_4$ is entirely $(\Delta + 3)$-colorable. In 2012

Wang, Mao, and Miao answered in the affirmative for simple, plane graph G with $\Delta \geq 8$. In this presentation, we will dicuss the result that every plane multigraph with $\Delta = 7$, no loops, no 2-faces, and no 3-faces sharing an edge is entirely $(\Delta + 3)$-colorable.

COLORING AROUND FACES TO COUNT DAISIES

Cat Myrant
Advisor: Dr. Oscar Levin
University of Colorado, Denver

A planar graph is one that can be drawn in the plane without edges crossing. However this embedding need not be unique. To distinguish between different embeddings of a single planar graph we introduce the face-wise chromatic number. This is analogous to the usual chromatic number, except now we require vertices incident to the same face to be colored distinctly. We will investigate the face-wise chromatic number for a particular class of graphs called daisy graphs. This will give insight into the behavior of the face-wise chromatic number and conversely suggests a way to classify different planar embeddings of these graphs.

ON THE 123-CONJECTURE FOR PRODUCTS

Michael Phillips[1]
Advisor: Dr. Cory Palmer[2]
[1]***University of Colorado, Denver***,
[2]***University of Montana***

A *k-edge-weighting* of a graph G is a mapping from the edges of G to $\{1, 2, \ldots, k\}$. For the primary conjecture, the *colour* of a vertex is defined to be the sum of the weights on all edges incident to that vertex. We say that a k-edge-weighting *induces a proper colouring* of $V(G)$ (or is *proper*) if for all pairs of adjacent vertices $u, v \in V(G)$, the colour of u is different than the colour of v. The 123-Conjecture, coined by Karoński, Łuczak, and Thomason in 2004, purports that for any connected graph G of order at least 3 there exists a proper 3-edge-weighting when the function used for vertex colouring is summation. Though this problem has yet to be solved, great strides have been made in recent years. In addition, variations on this problem have been posed; it is with these problems that we concern ourselves.

The variation featured in this talk is to redefine the colour of a vertex to be the *product* of the weights on all edges incident to said vertex. We show that K_n is the only complete multipartite graph on n vertices for which there does not exist a proper 2-edge-weighting when colouring with products. We also show that there exists a proper 2-edge-weighting when colouring with sums for complete k-partite graphs which have fewer than $k/2$ parts of size 1.

MORNING SESSION II

A PDE APPROACH TO PATTERN DETECTION IN WILDLAND FIRE RESOURCE ALLOCATION DATA

Alex Masarie[1]
Advisor: Yu Wei[1]
Committee Members: Matthew Thompson[2]**,**
Mike Bevers[2]**, & Iuliana Oprea**[1]
[1]***Colorado State University***
[2]***Rocky Mountain Research Station,***
US Forest Service

Interactions between humans and wildland fire generate complex management problems with environmental and social dimensions. We modeled the national allocation process, in which specialized fire resources are mobilized around the country to suppress and monitor incidents, using core PDE concepts of advection, diffusion, and reaction. We applied the model to detect how demand for crews and engines during 2011-2015 responded to regional management factors related to ongoing fire activity, suppression resource use, fire weather, expenditures, accessibility, and population density. This presentation will demonstrate how finite difference forward and inverse methods were used to characterize variation in demand. We will explain how node-based stenciling techniques led to stable numeric schemes and helped connect demand response to fire risk information. We will describe lessons learned from creating a structured mesh of the continental United States, balancing prediction and assimilation with time-stepping schemes, and reducing the underlying functional spaces into tractable parameter estimation subspaces. Results from computational experiments focused on pressure-fitting, spatial gradient estimation, and short-term prediction will be discussed in terms of efficient allocation of the national supply. As a reverse engineering of national demand dynamics this work underlines advantages and disadvantages of applying a PDE process model to a human decision-making problem.

ESTIMATING THE IGNITION POINT OF WILDFIRES

James Haley
Advisor: Jan Mandel
University of Colorado, Denver

Using satellite detections of fires, we can estimate the ignition point of a wildfire. In this talk we look at how a few early detections of a fire can be used to generate a set of plausible ignition points from which we can simulate a wildfire. After running the simulations, we then pick a best guess ignition point by evaluating the data likelihood additional satellite observations.

SMART ELECTRIC VEHICLE CHARGING UNDER UNCERTAINTY

Devon Sigler
Advisors: Jiucai Zhang, Wesley Jones, & Alex Engau
University of Colorado, Denver

The power grid can be stressed significantly by many electric vehicles charging. A purposed solution to address the added stress is to have electric vehicles that participate in demand response by charging in a price responsive manner to market pricing signals. We present a mathematical formulation of a price responsive stochastic EV charging controller. This controller makes charging decisions to minimize charging costs based on price signals from the independent system operator, while also minimizing the range anxiety experienced by the driver when low states of battery charge occur. Since future driving schedules and electricity prices are uncertain, optimization techniques are used to make the controller's charging decisions robust to these uncertainties. Results from testing the performance of the controller under simulation are presented.

MATHEMATICAL MODELING OF THE HPA AXIS

Justin Garrish[1]
Advisor: Johnny Ottesen[2] Collaborators: Jasdeep Pannu[3], Pei Pei[4], Gregory Moses[5], & Julia Walk[6]
[1]***University of Colorado, Colorado Springs***, [2]***Roskilde University***, [3]***Lamar University***, [4]***Otterbein University***, [5]***University of Ohio***, [6]***Concordia College***

The hypothalamus, pituitary gland, and adrenal glands form what is known as the HPA axis. These three organs act to modulate stress through the release of hormones, and the rates at which these hormones are released at a given time depend on their concentrations in the bloodstream. We can monitor the amounts of some of these hormones, for instance, adrenocorticotropic hormone (ACTH), which is secreted by the pineal gland to stimulate the adrenal cortex, and cortisol (CORT) which is subsequently released by the adrenal glands and suppresses the release of corticotropin releasing hormone (CRH) by the hypothalamus. The biological model of the HPA axis allows for the development of differential equations to model the endocrine dynamics, and these data allow the models to be calibrated. We discuss the development of such a model, its implications, and its limitations. Further, a hypothetical extended model is discussed, where the current model is coupled with some underlying intracelluar dynamics. Once the model is established, we discuss the goal of parameter estimation in the coupled system, and the effectiveness of various optimization strategies.

CONSTRUCTION OF MINIMAL AUTOMATA FOR GENERALIZED LANGUAGES Á LA AHO-CORASICK

Ian Char
Advisors: Manuel Lladser
University of Colorado, Boulder

In the world of bioinformatics, a significant problem is to recognize certain patterns, or *biological motifs*, in a DNA sequence. A standard approach for finding such motifs is to employ a graph-like structure called a *deterministic finite automaton* (DFA). However, the practicality of this approach relies on the DFA having manageable size. To address this problem, we focus on a class of biological motifs known as *generalized strings.* Given such a string, we present an algorithm that constructs its associated minimal DFA. This construction is direct (i.e. our DFA will never grow larger than the minimal automaton) and allows one to easily encode the transitions for further compression. This project has been partially funded by EXTREEMS-QEDNSF grant 1407340.

A ZIPPER MODEL FOR CRISPR-Cas9 BINDING SYSTEMS

Jonathan W. Lavington
Advisors: Manuel Lladser
University of Colorado, Boulder

CRISPR-Cas9 is a genetic engineering technique discovered in the early 2000s that researchers use to make targeted changes to an organism?s DNA. This technique has created new avenues for the manipulation of microbial metabolisms (e.g. for ethanol production), and the possibility of effective gene therapy. Unfortunately, while great strides have been made over the last decade, this method is still prone to inaccuracies, often generating sub-optimal gene editing. CRISPR-Cas9 is an enzyme that recognizes specific sequences of DNA, or variations thereof, which it targets and removes from the host genome with varying efficiencies. The efficiency with which it cleaves a specific sequence depends on the number and location of mismatches between that sequence and the desired target. In this talk, we present a differential equations model combined with a discrete Markov chain model to predict the efficiency of CRISPR-Cas9 based on various factors. Using global optimization algorithms, we fit experimental results by finding optimal parameters for our physico- probabilistic model. Using these parameters we can now predict off-target effects and guide the design of future experiments. This work is in collaboration with K. Tarasava at the Gill Lab (ChBE) at CU-Boulder, and has been partially funded by the EXTREEMS-QED NSF grant 1407340.

MORNING SESSION III

ACTIVE SUBSPACES IN ENERGY MODELING

Shannon Grubb
Advisor: Paul Constantine
Colorado School of Mines

Using JEPlus whole building simulations, NIST has created a way of looking at the life cycle cost (LCC) of residential buildings, specifically looking at the economic benefits of increasing the energy efficiency of new construction. With the many possible inputs, both economic and construction based, running and storing all of the data for public use rapidly becomes a burden. Here, we use active subspaces to identify a lower dimensional structure underneath the model that allows almost equivalent information with fewer data points stored. This also provides insight behind what energy efficiency measures are most valuable economically.

INVERSE REGRESSION FOR RIDGE RECOVERY

Andrew Glaws
Advisor: Paul Constantine
Collaborator: Dennis Cook
Colorado School of Mines

We investigate the application of sufficient dimension reduction (SDR) to a deterministic function of several variables. In this context, SDR provides a framework for ridge recovery. A scalar function of m inputs is referred to as a ridge function when it may be expressed as a function of $n < m$ linear combinations of the original inputs. The ridge directions contain the weights of these linear combinations. Ridge recovery methods attempt to discover the ridge subspace spanned by the ridge directions. Many ridge recovery methods, such as active subspaces, require the gradient of the function relative to the original inputs. Such gradient information may be unavailable, and methods for approximating the gradient (e.g., finite differences) might be prohibitively expensive.

Sufficient dimension reduction (SDR) provides a theoretical framework for dimension reduction in statistical regression. In contrast to deterministic functions, regression contains noise in the relationship between the predictors and response?i.e., multiple values of y are possible for a single x. SDR defines a subspace of the predictors in which no loss of statistical information occurs to the conditional random variable $y|x$. This is accomplished by enforcing conditional independence of the response and predictors given the reduced predictors. We prove that, given the deterministic relationship $y = f(x)$, the dimension reduction subspaces from SDR are equivalent to the ridge subspaces from ridge recovery.

We study two inverse regression methods for SDR-sliced inverse regression (SIR) and sliced average variance estimation (SAVE)-that use predictor/response pairs to search for the ridge subspace. These methods require no gradient information, which makes them promising candidates for gradient-free approaches to ridge recovery. We provide convergence results for these algorithms for solving the ridge recovery problem, and we demonstrate the methods on simple numerical test problems.

EXPLORING ACTIVE SUBSPACES IN NEURAL NETWORK COST FUNCTIONS

Jonathan Helland
Advisor: Paul Constantine
Colorado School of Mines

Neural networks are a popular method in supervised machine learning for solving classification problems. To solve a classification problem, a neural network must first be trained on data. To do this, a measure of error called a cost function is minimized via a heuristic method like gradient descent. However, neural networks commonly seek to classify high dimensional data, which requires that many parameters be learned. This process is computationally expensive and gradient descent is often caught by local minima in the network's cost function that may be far from the global minimum. It is desirable, then, to work with an exploitable low-dimensional subspace of the high-dimensional parameter space. We seek this dimensionality reduction by searching for active subspaces in neural network cost functions.

ICM PROBLEM F: MIGRATION TO MARS: UTOPIAN WORKFORCE OF THE 2100 URBAN SOCIETY

Ellen Considine, Suyog Soti, & Emily Webb
University of Colorado, Boulder

For centuries, humanity has dreamed of colonizing another planet. In less than five years, ten thousand people will become the first permanent inhabitants of Mars. While the preparation of infrastructure in the colony - including shelter from radiation and the elements and the provision of food and water - is already underway, little thought has yet been given to the political-economic system that will govern this future society. This report finds that the institution of a universal basic income on Mars will allow for the creation of a self-sustaining society whose priorities include the well-being of all its citizens, a high caliber of education, and social equality.

To develop these societal priorities into a working system, our team built a model to optimize the combination of Gross Domestic Product (GDP) and subjective well-being (happiness) of the Martian colony with variable income tax rates. After running a program to find the constants for our tax equation that optimized GDP and Happiness while keeping personal income tax rates under 60% (the top income tax rate on Earth), we discovered that the relationship between our GDP and Happiness scores reflects that described in prominent real-world research on the same correlation. Extending the model to accommodate a more diversified (global) population revealed that other common evaluations, such as the Human Development Indices, also reflect well on the accuracy and progressive nature of our model. The results of our model verify that it is possible to establish a sustainable society with both a basic income and universal higher education, which we recommend be initiated on Mars.

MULTI-OBJECTIVE SHAPE OPTIMIZATION OF TRANSONIC AIRFOILS USING ACTIVE SUBSPACES

Zach Grey
Advisor: Paul Constantine
Colorado School of Mines

Multi-objective shape optimization problems involve parameterizing a shape and modifying the shape to minimize a set of objectives. However, as in the case of transonic flow regimes, the computational cost of approximating a partial differential equation to compute quantities of interest is prohibitive. Response surfaces and surrogate modeling methods enable computationally efficient approximations of quantities of interest. These surrogate methods benefit from dimension reduction of the parameter space. A reduced dimension active subspace enables dimension reduction of the parameter space based on a particular quantity of interest. Changing parameters in the active subspace change that quantities more, on average, than changing parameters orthogonally to the active subspace. The active subspace also characterizes sensitivity information for each quantity of interest. We quantify the active subspaces present in transonic lift and drag coefficients and relate the subspaces to physical characteristics of the shape parameterization. Using the resulting subspaces, we characterize continuous shape perturbations over a Pareto frontier.

ANALYZING FCQ RESULTS USING ADVANCED DATA ANALYTICS

Aaron Nielsen, Haoyue Zhang, & Lilong Wang
University of Colorado, Denver

In this talk, we will be using a variety of statistical methods to analyze faculty/course questionnaire (FCQ) results. In particular, we will be presenting a thorough exploratory data analysis of FCQ scores in the Mathematics department over the past two years. We will also be investigating potential factors that affect overall course score ratings using weighted least squares regression. Finally, we will use clustering analysis in an attempt to find common factors that lead to higher FCQ scores.

AFTERNOON SESSION I

NUMERICAL STUDY OF STEM IN (3142) KP LINE SOLITON SOLUTION

Tommy McDowell
Advisor: Dr. Sarbarish Chakravarty
Collaborator: Michelle Osborne
University of Colorado, Colorado Springs

This talk will present the general structure of a multicriteria optimization problem under uncertainty and introduce some notions The Kadomtsev-Petviashvilli (KP) equation admits an important class of solitary wave solutions that are regular, non-decaying, and localized along distinct lines in the xy-plane. The line-soliton solutions of KP have been studied extensively in recent years. These solutions consist of an arbitrary number of line solitons in the far-field region and a complex, web-like interaction pattern of intermediate solitons in the near-field region. Such wave patterns arise in nature as beach waves, and have also been reproduced in laboratory settings in water tank experiments.

Numerically we have studied different initial conditions to see which line-soliton solutions of KP they converge to. In particular, the (3142)-type soliton solution of KP has the rather unique feature that it consists of a high amplitude intermediate soliton we call the stem. Two different initial conditions evolve over time to form this (3142)-type soliton solution of KP. These are the V Shape and Bow Shape initial conditions. Here we present an analysis of the aforementioned stem for both initial conditions.

NON-CLASSICAL DISPERSIVE SHOCK WAVES IN SHALLOW WATER

Patrick Sprenger
Advisors: Mark A. Hoefer
University of Colorado, Boulder

A classical model for shallow water waves with strong surface tension is the Kawahara equation, which is the Korteweg-de Vries (KdV) including a fifth order derivative term. A particular problem of interest to these types of equations is step initial data, known as the Riemann problem, which results in a shock in finite time. Unlike classical shock waves, where a discontinuity is resolved by dissipation, the dispersive regularization results in the discontinuity resolved as a dispersive shock wave (DSW). When parameter choices result in non-convex dispersion, three distinct dynamic regimes are observed that can be characterized solely by the amplitude of the initial step. These distinct regimes are studied numerically and asymptotically with interesting new results. A table top experiment in which these structures can be observed will then be discussed and preliminary experimental observations will be shown.

PERIMETER DYNAMICS OF A TWO-DIMENSIONAL, CIRCULAR SOLITON IN A MAGNETIC SYSTEM

Maximilian Ruth
Advisors: Mark A. Hoefer & Ezio Iacocca
University of Colorado, Boulder

In the continuum approximation, the dynamics of a thin-film ferromagnet can be modeled by a nonlinear partial differential equation known as the Landau-Lifshitz equation. This equation admits a one-parameter family of stationary, two-dimensional (2D) solitary wave solutions known as droplet solitary waves or solitons (droplet hereafter). Interest in droplets has heightened of late due to its observation in certain magnetic devices and its potential for technological applications. In the idealized setting of a lossless magnet, the droplet is a stable, localized, circular wave structure that is dynamically precessing with a single frequency that is inversely proportional to the droplet radius. However, real magnetic devices where droplets have been observed are not idealized, subject to various perturbations such as damping or loss. In addition, numerical simulations have revealed dynamic, perimeter deformations to the droplet's circular structure. This talk will present a mathematical approach to understand how the droplet's perimeter evolves due to perturbations.

Two methods are used to investigate these perturbations, in order of increasing complexity and power: i) an effective Lagrangian approach and ii) perturbation theory applied to the differential geometry of the droplet's perimeter. Both methods effectively reduce the problem of characterizing the 2D droplet into solving a 1D system, as the dynamics of the droplet are localized to the perimeter. Remarkably, the resulting nonlinear dynamics of the droplet are described by linear partial differential equations for the droplet radius and a phase angle. The general solution is determined via Fourier series.

The differential geometry approach is more general. For instance, magnetic damping, a physical perturbation of fundamental importance, leads to linear loss terms in the partial differential equation for the perimeter dynamics. It is expected that other physically relevant perturbations can be incorporated as well, in order to obtain a mathematical description of droplets in experiments.

The presentation will cover both methods for determining the droplet's perimeter dynamical equation. The perturbative effect of magnetic damping will be described and favorable comparisons between the approximate, analytical solutions and full numerical simulations of the Landau-Lifshitz equation will be shown.

THE LOWEST ORDER WEAK GALERKIN SOLVER FOR THE DARCY EQUATION ON HEXAHEDRAL MESHES

Graham Harper[1]
Advisor: Dr. James Liu[1]
Collaborator: Dr. Bin Zheng[2]
[1]***Colorado State University,***
[2] ***Pacific Northwest National Laboratory***

We present the lowest order weak Galerkin (WG) finite element method for solving the Darcy equation on general hexahedral meshes. The lowest order WG method uses constant unknowns for pressure inside the elements and constant unknowns on the faces, but defines the weak gradients of the basis functions in local Raviart-Thomas spaces. The solver has easy implementation, and it has several numerically favorable properties, including local mass conservation and continuity of normal flux across elements, regardless of mesh quality. When the mesh is asymptotically parallelopiped, this solver exhibits optimal order convergence in pressure, velocity, and flux, as demonstrated by numerical results on perturbed brick meshes and cylindrical meshes.

AFTERNOON SESSION II

A VERY BRIEF INTRODUCTION TO QUANTUM COMPUTING

Jaden Pieper
Advisors: Manuel Lladser
University of Colorado, Boulder

As industry continues to inspire great growth in the research and development of physical quantum computers, it is increasingly worthwhile to familiarize oneself with the computational theory of this new and exciting field. In this talk, I will describe the theory of some of the foundational concepts of quantum computing, such as qubits and quantum gates. I will then briefly explore two quantum algorithms: Deutsch's and Grover's algorithms. Deutsch's algorithm lets one determine if an unknown function $f : \{0,1\} \to \{0,1\}$ is or is not a constant, with just a single evaluation of the function. On the other hand, Grover's algorithm finds a needle in a haystack, that is, given some unsorted list of N items, it can identify a target item in $O(\sqrt{N})$ instead of the $O(N)$ iterations required by classical computing. This project has been partially funded by EXTREEMS-QED NSF grant 1407340.

A KERNEL-BASED LAGRANGIAN METHOD FOR IMPERFECTLY-MIXED CHEMICAL REACTIONS

Michael J. Schmidt
Advisors: David Benson
& Stephen Pankavich
Colorado School of Mines

Lagrangian particle tracking methods are highly useful tools for simulating reactive transport, especially under circumstances when imperfect mixing across a range of scales would require a high level of spatial discretization in an Eulerian, grid-based method. Previous work has shown that reactive particle tracking methods using point-particles (represented by Dirac delta distributions) are an effective method for simulating highly heterogeneous conditions, in both velocities and chemical compositions.

The number of particles needed in a simulation is directly related to the chemical smoothness (autocovariance of concentrations), i.e., the mixing condition. As the chemical conditions become smoother, larger number of particles (approaching infinity, for the perfectly-mixed case) are required to effectively simulate these conditions, pushing computation times higher. Here we show an approach that reduces the number of particles in a given simulation by imparting extra "mass" to each particle in the form of a Gaussian kernel representation. These kernels "smooth" the concentrations in a user-controlled manner, and simulate any desired level of mixing, while benefiting from faster computation times. Simulations using these kernel-based particle tracking methods are shown to converge to the point-particle method, as well as analytical solutions obtained from the underlying equations derived using a moment equation approach.

MANY EDGE-DISJOINT RAINBOW SPANNING TREES IN GENERAL GRAPHS

Lauren M. Nelsen
Advisor: Paul Horn
University of Denver

A rainbow spanning tree in an edge-colored graph is a spanning tree in which each edge is a different color. Carraher, Hartke, and Horn showed that for n and C large enough, if G is an edge-colored copy of K_n in which each color class has size at most $n/2$, then G has at least $\lfloor n/(C \log n) \rfloor$ edge-disjoint rainbow spanning trees. Here we strengthen this result by showing that if G is *any* edge-colored graph with n vertices in which each color appears on at most $\delta \cdot \lambda_1/2$ edges, where $\delta \geq C \log n$ for n and C sufficiently large and λ_1 is the second-smallest eigenvalue of the normalized Laplacian matrix of G, then G contains at least $\left\lfloor \frac{\delta \cdot \lambda_1}{C \log n} \right\rfloor$ edge-disjoint rainbow spanning trees.

ERDŐS-SZEKERES ONLINE

Luke L. Nelsen
University of Colorado, Denver

In 1935, Erdős and Szekeres proved that $(m-1)(k-1)+1$ is the minimum number of points in the plane (ordered by their x-coordinates) which definitely contain an increasing (also ordered by y-coordinates) subset of m points or a decreasing subset of k points. We consider their result from an online game perspective: Let points be determined one by one by player A first determining the x-coordinate and then player B determining the y-coordinate. What is the minimum number of points such that player A can force an increasing subset of m points or a decreasing subset of k points? In this talk, we discuss the distinction between the original setting from this new one and present some small results. Thanks to the 2016 GRWC workshop, work on this question is underway jointly with Kirk Boyer, Lauren M. Nelsen, Florian Pfender, Lizard Reiland and Ryan Solava.

AFTERNOON SESSION III

EDUCATIONAL APPLICATIONS OF UNDERGRADUATE RESEARCH THROUGH COMPILING A DATABASE AND SPECIAL TOPICS IN OPTIMIZATION RESEARCH

Christina Ebben & Keegan Schmitt
Advisor: Steffen Borgwardt
University of Colorado, Denver

The CU Denver Optimization Wiki is a resource for understanding introductory topics in mathematical programming. As a group of undergraduate students working on separate topics, the wiki serves as collaborative report on our research. The finished product will be an easy to understand online database for educators and students in the field of optimization research. We provide an overview of the project and problems encountered thus far as well as future directions. We will also cover interesting topics from our research including various methods for formulating and understanding a dual program as well as an introduction to Lagrangian Duality and Lagrangian multipliers.

THE MATHEMATICS OF GOSSIP

Izabel Aguiar, Jessica Deters,
& Jacquie Feuerborn
Colorado School of Mines

In this independent project we develop a numerical model to investigate the spread of rumors, lies, and gossip throughout a community using an adapted susceptible-infected-recovered (SIR) dynamical system. We employ the model to study parameter dependence and system properties. Through our research we explore and illuminate how mathematics can help us better understand sociological ideas.