18th Annual SIAM Front Range Applied Mathematics Student Conference

March 12th, 2022

For the remote FRAMSC experience, there will be a single Zoom meeting with breakout rooms corresponding to the physical room numbers for the different sessions. The Zoom link is 982 2251 8116. We have two morning sessions and two afternoon sessions.

Morning Session I: 9:00 - 11:00

Student Commons Building, Room 4125

9:00 - 9:20	Eric Culver *University of Colorado, Denver*	Computational Methods Applied to Graph Coloring Problems
9:25 - 9:45	Alex Paradise *University of Colorado, Boulder*	Toward the Metric Dimension of Jaccard Metric Spaces
9:50 - 10:10	Rebecca Robinson *University of Colorado, Denver*	Star Chromatic Index of Subcubic Graphs
10:15 - 10:35	Nicholas Crawford *University of Colorado, Denver*	3-Colorability of Graphs with Minimum Degree at Least 6
10:40 - 11:00	Daniel Ferguson *University of Colorado, Boulder*	Computation of the Sample Frechet Mean for Sets of Large Graphs with Applications to Regression

Morning Session II: 9:00 - 11:00

Student Commons Building, Room 4017

9:00 - 9:20	Van Hovenga *University of Colorado, Colorado Springs*	Maximum Likelihood Estimation of the Three-Dimensional Structure of Chromosomes from Hi-C Data
9:25 - 9:45	Kirana Bergstrom *University of Colorado, Denver*	A Constrained Quadratic Optimization Approach to Data-Consistent Inversion
9:50 - 10:10	Brian Cain *University of Colorado, Boulder*	Automatic Melanoma Detection via Skin Lesion Images
10:15 - 10:35	Drew Horton *University of Colorado, Denver*	Hungry for Equality: Fighting Food Deserts with Optimization
10:40 - 11:00	Nicholas Weaver *University of Colorado, Denver*	Foodomics: Understanding the Impact of Different Foods on Metabolism and Health

University of Colorado Denver
CU IN THE CITY

We gratefully acknowledge funding support from CU Denver Dept. of Mathematical and Statistical Sciences and the J.R. Woodhull/Logicon Teaching Professor of Applied Mathematics at CU Boulder.

The industry panel and plenary address will take place in the Student Commons Building, Room 2600.

Industry Panel: 11:15 - 12:00

Panelists:

- Christina Kumler, Research Associate
  (*CIRES/NOAA Global Systems Laboratory*)
- Maya Bam, Senior Research Scientist
  (*General Motors*)
- Ankan Kumar, Senior Machine Learning Engineer
  (*Amazon*)

Lunch Break: 12:00 - 12:40

Plenary Address: 12:40 - 1:40

"Statistics in the News: How statistical thinking can help us be better consumers of the numbers, data, and statistics all around us"

Dr. Audrey Hendricks. *Dept. of Mathematical and Statistical Sciences*, *University of Colorado, Denver.*

A group picture will be taken immediately following the plenary address.

Afternoon Session I: 1:50 - 3:25

Student Commons Building, Room 4125,

(Mathematical Contest in Modeling Presentations)

1:50 - 2:10	Wuyan Wang *University of Colorado, Boulder*	Evolution of partial, truncated and bent deep water internal wave solitons
2:15 - 2:35	Wenqian Xu *University of Colorado, Boulder*	From Soliton and Periodic Traveling Waves to Their Interactions: Theory and Experiment
2:40 -3:00	Ziqiang Li *University of Wyoming*	Barycentric Finite Elements for Spherical Triangles
3:05 - 3:25	Alexey Yermakov *University of Colorado, Boulder*	Imaging Three-Dimensional Surface Waves Using a Synthetic Schlieren Method

Afternoon Session II: 1:50 - 3:50

Student Commons Building, Room 4017

1:50 - 2:10	Haoru Yang, Yousheng Tang, and Joshua McFeeters *Colorado College*	Modeling Forest Carbon Sequestration
2:15 - 2:35	Zhengfeng Zheng, Yuzhou Shen *University of Colorado, Boulder*	The Magic Weapon to Win The Olympics Cycling Road Time Trial
2:40 -3:00	Aidan Rooney, Rachel Johnson, Jaden Mascarenhas *University of Colorado, Colorado Springs*	Carbon Sequestration in Forests - COMAP ICM Problem E Report
3:05 - 3:25	Jingyi Liu and Esa Chen *Colorado College*	Winning Strategy for Individual Time Trial Contest: Maxed Out at the Beginning
3:30 - 3:50	Davidson Cheng and Yinting Zhong *Colorado College*	An Evolutionary Economic Model for Predicting the Equity of Asteroid Mining

Plenary Speaker

12:40 - 1:40

Statistics in the News: How statistical thinking can help us be better consumers of the numbers, data, and statistics all around us

Audrey Hendricks

Associate Professor, Department of Mathematical and Statistical Sciences
University of Colorado, Denver

New statistics and machine learning models hold broad excitement in many areas including personalized medicine, targeted advertising, self- driving cars, and farming. Indeed, businesses now tout AI divisions in advertising campaigns. But as can often happen with new, exciting technology, missteps have occurred. Here, I show how a focus on foundational statistics principles can help experts and the general public alike be more informed and critical consumers of statistics (and AI) in the news, journal articles, and our everyday life.

About the Speaker

Audrey Hendricks is an Associate Professor at University of Colorado at Denver in the Department of Mathematical and Statistical sciences with secondary appointments in Biostatistics and Informatics, the Colorado Center for Personalized Medicine, and the Human Medical Genetics and Genomics Program. She is passionate about data science literacy for all and removing barriers to learning. Dr. Hendricks's research focuses on collaborative applied projects and statistical method development to better understand the complex nature of human diseases and traits. Recently, this includes studies to elucidate biological mechanisms behind the relationship between food and health as well as the development of methods and user-friendly software to increase the utility and equity of publicly available genetic data, especially for diverse populations. She is proud to mentor many amazing undergraduate and graduate students working on statistical challenges in human genetics and genomics. You may learn more about Dr. Hendricks at https://audreyhendricks.com.

MORNING SESSION I

COMPUTATIONAL METHODS APPLIED TO GRAPH COLORING PROBLEMS

Eric Culver
University of Colorado, Denver

The discharging method is a common technique applied to coloring prob- lems on planar graphs. We discuss the main ideas of the discharging method and apply it to the square choosability number of planar graphs. A generalization of the usual vertex coloring, the square choosability number is the least size of lists of colors needed to be assigned to the vertices of a graph so that, no matter what colors are in the lists, every vertex can be colored with a color from its list and every two vertices at distance at most two from each other receive different colors. A key component of our approach is the use of computers to handle the complexity of parts of the proof.

A TOWARD THE METRIC DIMENSION OF JACCARD METRIC SPACES

Alex Paradise
University of Colorado, Boulder

A subset of points in a metric space is called "resolving" when any other point is uniquely determined by its distance to the points in the set. The "metric dimension" of the (metric) space is defined as the size of its smallest resolving set.

Given a non-empty set X, let 2^X denote its powerset (i.e., the set of all possible subsets of X). The Jaccard distance between $A, B \in 2^X$ is defined as $d(A, B) := 1 - |A \cap B|/|A \cup B|$. d is a metric on 2^X. In this talk, I will present various preliminary results that hint at the metric dimension of the power set of X with respect to the metric d. A potential application of this to represent tweets as low-dimensional numerical vectors will also be discussed.

STAR CHROMATIC INDEX OF SUBCUBIC GRAPHS

Rebecca Robinson
University of Colorado, Denver

A graph G is composed of vertices connected by edges. A subcubic graph is a graph where each vertex is connected to at most three other vertices. We can color the edges of the graph such that no two edges that share a vertex obtain the same color, and such a coloring is called a proper edge coloring. A star edge coloring of a graph G is a proper edge coloring such that no paths or cycles of length four are colored with only two colors. The star chromatic index of a graph is the minimum number of colors needed to give a star edge coloring of G. It has been proven that the star chromatic index of a subcubic graph is at most 7, but it has been conjectured that this bound can be reduced to 6. We show an integer program that has been used to experimentally verify this conjecture for subcubic graphs on at most 19 vertices. We also discuss some partial results we have towards a possible discharging proof of the conjecture.

Abstract

3-COLORABILTY OF GRAPHS WITH MINIMUM DEGREE AT LEAST 6

Nicholas Crawford
University of Colorado, Denver

Graph coloring started as a formal problem back in the 1850's when Augustus De Morgan sent a letter to William Rowan Hamilton asking about the now famous 4-coloring problem. Vertex coloring has since been explored immensely and has various applications such as computer programming and route making. In 2013 Beigel and Eppstein proved the fastest known algorithm for determining if a graph is 3-colorable. For this presentation we will improve upon their algorithm for graphs with restrictions on the minimum degree of a vertex.

COMPUTATION OF THE SAMPLE FRECHET MEAN FOR SETS OF LARGE GRAPHS WITH APPLICATIONS TO REGRESSION

Daniel Ferguson
University of Colorado, Boulder

To characterize the location (mean, median) of a set of graphs, one needs a notion of centrality that is adapted to metric spaces, since graph sets are not Euclidean spaces. A standard approach is to consider the Frechet mean. In this work, we equip a set of graphs with the pseudometric defined by the 2-norm between the eigenvalues of their respective adjacency matrix. Unlike the edit distance, this pseudometric reveals structural changes at multiple scales, and is well adapted to studying various statistical problems for graph-valued data. We describe an algorithm to compute an approximation to the sample Frechet mean of a set of undirected unweighted graphs with a fixed size using this pseudometric.

MORNING SESSION II

MAXIMUM LIKELIHOOD ESTIMATION OF THE THREE-DIMENSIONAL STRUCTURE OF CHROMOSOMES FROM HI-C DATA

Van Hovenga
University of Colorado, Colorado Springs

The three-dimensional (3D) structure of chromosomes plays a large role in many genomic functions, such as DNA replication, DNA repair, DNA translocation, and gene regulation. Hi-C is a technique to quantify the micro-structures of genomes by measuring contacts between genomic loci. In this talk, we formulate a method for predicting the 3D structure of chromosomes from Hi-C data. Our method relies on maximizing a scaled Gaussian likelihood over the distribution of loci distances inferred from Hi-C contacts. We employ gradient ascent along with a novel curriculum training strategy to maximize this likelihood. We validate our method on the GM12878 cell line and compare with existing techniques in the literature.

A CONSTRAINED QUADRATIC OPTIMIZATION APPROACH TO DATA-CONSISTENT INVERSION

Kirana Bergstrom
University of Colorado, Denver

Data-consistent inversion involves the characterization of a probability measure on the input parameters of a computational model whose push-forward matches an observed probability measure on specified quantities of interest associated with model outputs. Such a solution is formally defined by the pullback of the observed probability measure and is therefore referred to as a data-consistent solution. It is possible to quantitatively characterize and approximate a probability measure in several ways. Previous approaches for approximating data-consistent solutions relied upon density estimation or set/event approximations. Such approaches are challenging to implement in scenarios where the number of either simulated or observational data are limited. The proposed research focuses on the construction and analysis of optimally weighted empirical distribution functions to approximate the data-consistent solution. This research extends previous work that analyzed a constrained quadratic optimization approach for defining weighted empirical distribution functions that solve a "forward" change-of-measure problem. We propose fundamental algorithmic and theoretical extensions of the original optimization approach to generalize the applicability to data-consistent inverse problems. Preliminary analysis suggests fundamental measure-theoretic approaches and testable assumptions on utilized optimization codebases are critical to this analysis.

AUTOMATIC MELANOMA DETECTION VIA SKIN LESION IMAGES

Brian Cain
University of Colorado, Boulder

Melanoma skin cancer is uncommon, yet it accounts for most skin cancer deaths. In particular, its automatic detection using dermoscopic images could facilitate its early detection and reduce its mortality. State of the art methods for automatic detection mostly rely on neural networks but fall short in two main areas:

(i) training datasets are predominantly from people with light skin tones, introducing biases for groups with darker ones; and

(ii) neural networks provide a lack of interpretable results for medical practitioners.

This talk presents various hand-crafted features extracted from images of skin lesions to overcome these issues. These features are motivated by expert knowledge, the so-called ABCDE's of melanoma, and may reduce training bias while increasing interpretability.

HUNGRY FOR EQUALITY: FIGHTING FOOD DESERTS WITH OPTIMIZATION

Drew Horton
University of Colorado, Denver

Food deserts are a form of food insecurity related to a lack of access to healthy, fresh, and affordable food. According to the United States Department of Agriculture (USDA), 13.7 million households in the U.S. experienced food insecurity in 2019. This problem has only been exacerbated by the ongoing COVID-19 pandemic, and disproportionately affects marginalized communities.

In one traditional approach where we seek to minimize the expected distance of the population to grocery stores, the worst-off members in our communities tend to be ignored in the solution as outliers. To address these food insecurities, and the existing inequities, we demonstrate how the Kolm-Pollak equally-distributed equivalent function (EDE) can be minimized over a facility location integer program to minimize not only distance but also the inequality of the distribution. The EDE is a nonlinear function making the problem computationally harder than the traditional linear model; therefore we discuss various ways to approach the optimization including a piecewise-linear under-estimator of the model. We present results demonstrating how our model works on real-world data to produce an optimal distribution of grocery store locations in New Orleans. In minimizing the inequality, we are ensuring that we are prioritizing relief, increasing food access, in disproportionately affected marginalized communities.

FOODOMICS: UNDERSTANDING THE IMPACT OF DIFFERENT FOODS ON METABOLISM AND HEALTH

Nicholas Weaver
University of Colorado, Denver

Foodomics is the study of foods and nutrition through the lens of chemistry and microbiology. Although we know the general health benefits, or consequences, of most foods, little is known about the underlying chemistry involved in metabolism. In a controlled feeding study conducted at Purdue University, 41 subjects with unhealthy baseline diets enrolled in a Mediterranean diet intervention study. Health indicators, such as cholesterol and blood pressure, and plasma samples were gathered at 4 time points from each participant. Untargeted metabolomics were performed on all plasma samples and representative food pieces to construct the chemical signatures of each person and food in the study. Here we focus on a novel statistical framework for tracking these chemical signatures from foods to human plasma through a diet intervention study. The statistical framework results in the identification of putative health-driven biomarkers that provide a potential explanation for how the Mediterranean diet decreases cholesterol levels in the study participants.

AFTERNOON SESSION I

EVOLUTION OF PARTIAL, TRUNCATED AND BENT DEEP WATER INTERNAL WAVE SOLITONS

Wuyan Wang
University of Colorado, Boulder

Soliton in (2+1)-dimension dispersive models is studied. The evolution of partial, initially truncated, and bent-stem solitons for the 2D Benjamin-Ono (2DBOII) equation modeling deep water internal waves is analyzed using modulation theory. The evolution can be described by a hyperbolic system of equations for the amplitude and slope of the modulated line soliton. This work studies the long-term behavior of the interaction of simple wave solutions of the modulation equations resulting from the truncated and bent-stem initial conditions. The evolution of these waves resembles the behavior of confined internal waves that experience a sudden change in their boundary conditions. In the case of bent-stem solitons, a bifurcation is found in the boundedness of the interaction region, which depends on the initial soliton amplitude and corresponds to different long term behavior.

FROM SOLITON AND PERIODIC TRAVELING WAVES TO THEIR INTERACTIONS: THEORY AND EXPERIMENT

Wenqian Xu
University of Colorado, Boulder

The Korteweg-de Vries (KdV) equation is a canonical mathematical model of nonlinear waves, modelling many physical systems including shallow water waves. Soliton and periodic (cnoidal) traveling waves are well-known, well-studied solutions of the KdV equation and other model equations. However, soliton and cnoidal wave interactions have not received as much attention. Taking inspiration from the KdV equation, the soliton and periodic traveling wave solutions of the conduit equation, a mathematical model of conduit waves, are studied using a phase plane analysis. Then, the interactions of solitons and periodic traveling waves in a conduit fluid experiment are observed. The soliton-periodic wave interactions result in so-called breathers that can be of the elevation (bright) or depression (dark) variety.

BARYCENTRIC FINITE ELEMENTS FOR SPHERICAL TRIANGLES

Ziqiang Li
University of Wyoming

A spherical cartogram, also called an anamorphic globe, is a map on which each region is sized proportionally to some distribution enclosed therein. This talk recapitulates the physics-based idea behind the prior 2018 paper on "Diffusion-based Cartogram on Spheres", then proceeds to address a few shortcomings of this original implementation. To avoid these shortcomings altogether, this talk introduces an FEM-based reimplementation of the spherical cartogram using a new finite element designed for spherical triangles that uses a specific notion of barycentric coordinate systems. Some properties of this finite element will be discussed. The reimplementation of spherical cartograms will be discussed and showcased.

IMAGING THREE-DIMENSIONAL SURFACE WAVES USING A SYNTHETIC SCHLIEREN METHOD

Alexey Yermakov
University of Colorado, Boulder

The dispersive properties of gravity-capillary waves on the surface of a fluid will be studied quantitatively using the Free Surface–Synthetic Schlieren (FS-SS) method. To obtain three-dimensional surface representations of the waves, a series of image processing steps need to be implemented in conjunction with modern post-processing techniques to reduce error in the model. The FS-SS method itself relies on obtaining a displacement field of the fluid surface. This displacement field is obtained by having two images, both of which are from a camera positioned above a medium containing water on top of a randomized dot pattern. The reference image contains the undisturbed water surface and the second image contains a deformed fluid surface. A displacement field is constructed using Particle Image Velocimetry software by determining how the dots in the reference image were displaced in the second image due to light refracted by the fluid surface. Using FS-SS, the spatial gradient of the deformed surface height is obtained by its approximate proportionality to the displacement field. Finally, an inverse gradient operation can be used to extract the three- dimensional surface waves from the second image. Implementing this process required overcoming multiple obstacles that will be described.

AFTERNOON SESSION II

MODELING FOREST CARBON SEQUESTRATION

Haoru Yang, Yousheng Tang, Joshua McFeeters
Colorado College

The threats of climate change currently impose a drastic risk to modern society, and as such appropriate measures must be taken so as to prevent said risks. Anthropogenic emissions of carbon dioxide have only exacerbated the current threats that climate change and processes of global warming have imposed upon the current population. Remedies to climate change require immediate initiatives to reduce the emission of carbon dioxide into the environment. One such means presents itself via the removing of carbon dioxide from the environment via naturally occurring processes. This process is aptly known as carbon sequestration, which entails the storing of gaseous carbon into solid carbon pools. Carbon dioxide is naturally captured from the atmosphere through biological, chemical, and physical processes, which can be accelerated artificially through changes in land use.

In our project, we present a model that uses forests as a means of carbon sequestration and offer the most efficient way of accelerating the capture of carbon dioxide while conserving other economic values of a given forest.

For a given forest with specific types of trees in different age classes, our model can calculate the amount of carbon sequestration of this forest according to the total above-ground biomass (AGB) of trees. After showing a negative relationship between the number of trees and the growth rate of biomass, our model displays the result that regrowth of younger forests has potential to allow for more carbon sequestration over time when compared to the carbon sequestration benefits of not cutting forests at all. In the end, our model gives us an applicable solution that can achieve economic benefits by harvesting specific amount of different types of trees without reducing the level of carbon sequestration before.

THE MAGIC WEAPON TO WIN THE OLYMPICS CYCLING ROAD TIME TRIAL

Zhengfeng Zheng, Yuzhou Shen
University of Colorado, Boulder

The data analysis, modeling method, and its reasonable practical application are helpful to analyze the **energy distribution**, completion time, and external natural factors of cyclists in cycling road time trial, so as to optimize the energy distribution of cyclists in each section of the race to get the shortest completion time and get the best winning strategy.

Unlike other **short distance** racing games, or games where the total **energy consumption** is not considered, the **energy management** and reasonable distribution of the cyclists is the key to win the bicycle time trial game.

The relationship between the power output and the velocity of the cyclist in each stage under internal and external conditions has been analyzed and modeled in detail. A physical analysis for this sport is the prerequisite for an efficient model. Hence, the practical situation when a cyclist is riding on the road in **varied angles**, **curvatures**, and **weather situation**s has been considered. So, a **physical-based model** considering forces, and energy consumption for individual trials has been built. The model has a high **correlation** with the **path** of the course, so the accuracy of course path data affects the final result of the model. In order to gain the most effective result from our model, the 3-Dimension data of the course's path has been analyzed from two different real races to test performance of the model in different situations:

- The 2021 Olympic Time Trial in Tokyo, Japan.
- The 2021 UCI World Championship Time Trial in Flanders, Belgium.
- The Customized Virtual Courses.

Finally, a power distribution can be optimized by the model with discrete data. However, different accuracy map data were tested in the model, and the results shows great differences especially for low accuracy map data. Hence, this model is ill-conditioned in extreme low accuracy map data. The model will converge to a better and result with the improvement of the accuracy of map data.

CARBON SEQUESTRATION IN FORESTS - COMAP ICM PROBLEM E REPORT

Aidan Rooney, Rachel Johnson, Jaden Mascarenhas
University of Colorado, Colorado Springs

Climate change is already occurring and creating substantial global impacts. Even if we were to stop pumping greenhouse gasses into our atmosphere today, the existing gasses would continue to create climate impacts far into the future. Because of this, we need to put effort into actively removing and sequestering carbon from the atmosphere and reducing the greenhouse effect. One of the most efficient ways to accomplish this is by leveraging nature's most powerful carbon sequestration technology, trees. But, how can one determine how much carbon can be sequestered from forests, and how should we decide what forests to sequester carbon from? As participants of the Interdisciplinary Context in Modeling (problem E) we addressed these questions and provided an example newspaper article to address local concerns about sequestering carbon using the Pike National forest near Fairview, CO, USA.

WINNING STRATEGY FOR INDIVIDUAL TIME TRIAL CONTEST: MAXED OUT AT THE BEGINNING

Jingyi Liu and Esa Chen
Colorado College

As people from all over the world begin to involve in cycling races, there are many mythologies about how to optimize pacing strategy throughout the contest. In this paper, we built a Rider's Model with power curve and proposed an optimized strategy for a rider to employ in accordance with our model. First of all, we constructed a Fatigue model that analyzed the relationship between power and time with considerations of uphill and downhill, and the recovery rate of the rider. From there, we concluded that recovery is not efficient for rider. Then, we built our Rider's Model through force analysis and Pontryagin's maximum principle, with four parameters including air resistance, bending angle, friction coefficient, and effective mass. This model shows and proves that there are three main levels of power during the contest, with maximum and minimum power levels different among rider types, and an intermediate fixed singular power level which is independent from time and can be calculated by inputting parameter values. We showed that the rider should reach his or her maximum speed at the early stage of the contest, and then slowly decrease to a constant singular speed. When the rider gradually consumes all the energy, eventually the speed would drop to the critical power level (minimum power level). Secondly, in order to analyze the effects of different environmental factors and the slope of the road, we did a theoretical analysis on our model by testing through multiple weather factors and showing that our model is compatible with those factors. For the effect of the slope of the road, we analyzed how our power curve will be affected through deductions: it turns out that the power would rise through course of uphill and would drop through course of downhill. We applied this in modeling power curve for cyclists participated in Tokyo 2020 Olympics contest. Thirdly, we applied our model to two worldwide cycling contests: UCI and Olympics, and designed one special contest to run through our model. This special contest involves five sharp turns and one ”Leap of Faith” nontrivial road grade, which is a two-meter long leap from one side of the road to the other side. According to our model, the curve would have a discontinuous point. Last but not least, we extended our model to team contest for which we considered a group of six riders as two parts: two riders are designated to be windshield and the other four is the main riders. Our model concluded that the four cyclists would have extra energy at the end to make the final sprint.

AN EVOLUTIONARY ECONOMIC MODEL FOR PREDICTING THE EQUITY OF ASTEROID MINING

Davidson Cheng and Yinting Zhong
Colorado College

In this paper, we analyze the potential industry of asteroid mining along with its relevance to promoting global equity. We begin by dissecting equity into two aspects: equal entry opportunity and equal success opportunity, which means people with all backgrounds should have an equal chance in participating and becoming successful in various industries or fields. We then propose Kullback-Leibler divergence-based measurements for these two aspects and apply them to the summer Olympics over the years. Our results indicate the summer Olympics has been improving its geographical and gender inclusiveness over the past century.

Following our mathematical definition of equity, we envision the future for asteroid mining. We use the Tsiolkovsky rocket equation and planetary models to predict the cost-effectiveness of asteroid mining. By incorporating this into an evolutionary model, we calculate the relative level of participation in asteroid mining for different groups of people at different points in the future. Our model indicates that without external influence, the asteroid mining industry may reach a relatively equitable state about 60 years after it begins.

Finally, a “high-to-low” policy for promoting equity in asteroid mining is proposed. Analyses show that with the right level of assistance, our proposed policy will expedite the equity advancement process by 10 years while allowing the entire industry to reach higher welfare at the same time.