17th Annual SIAM Front Range Applied Mathematics Student Conference

March 13th, 2021

For our remote FRAMSC experience, we will have four Zoom rooms. There is one main hub Zoom room (Zoom ID: 981 3034 9510). This will be used for networking during coffee breaks, our industry panel and for the keynote speaker. Feel free to join the hub at anytime. The other three Zoom rooms are for presentations. We have three morning sessions and two afternoon sessions. The additional Zoom rooms correspond to these sessions. If you need any additional assistance, there should be someone available in the hub Zoom room to help.

Morning Session I: 9:00 - 11:35

Zoom ID: 955 4040 1849

9:00 - 9:20	Evan Gorman *University of Colorado, Boulder*	Low Dimensional Embeddability of Microbiome Data
9:25 - 9:45	Nicholas Weaver *University of Colorado, Denver*	Association Testing in Metabolomics Data with an Application to Complimentary Diets in Infants
9:50 - 10:10	Megan Duff *University of Colorado, Denver*	A Robust Analysis of Linear Methods for Phenotype Prediction
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Griffith Hampton *Colorado School of Mines*	Insulin Action Models Applied to Glycerol and Glucose Yield Different Dynamics
10:50 - 11:10	Nicholas W. Landry *University of Colorado, Boulder*	The Effect of a Viral Load Function on Epidemic Dynamics
11:15 - 11:35	Subekshya Bidari *University of Colorado, Boulder*	Hive Geometry Shapes the Recruitment Rate of Honeybee Colonies

Student Government Association
UNIVERSITY OF COLORADO DENVER

We gratefully acknowledge funding support from CU Denver Student Government Association.

Morning Session II: 9:00 - 11:35

Zoom ID: 922 5393 3279

9:00 - 9:20	Kirana Bergstrom *University of Colorado, Denver*	Estimating Observation-Consistent Solutions Using Weighted Empirical Distribution Functions
9:25 - 9:45	Richard Clancy *University of Colorado, Boulder*	Approximate Maximum Likelihood estimators for Linear Regression with Operator Uncertainty
9:50 - 10:10	Daniel Bielich *University of Colorado, Denver*	Delayed Classical Gram-Schmidt with Reorthogonalization (DCGS2) - Improved Gram-Schmidt Orthogonalization Algorithm for Parallel Computation
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	Lara Kassab *Colorado State University*	Semi-supervised Non-negative Matrix Factorization for Learning Tasks
10:50 - 11:10	Alyssa Newman *University of Colorado, Denver*	Analysis of the Asymptotics of the Lowest Unique Positive Integer Game
11:15 - 11:35	Zachary Sorenson *University of Colorado, Denver*	Applying the Cycle Cancelling Algorithm to the Traveling Salesman Problem

Morning Session III: 9:00 - 11:35

Zoom ID: 989 6084 6882

9:00 - 9:20	Zane Showalter-Castorena *University of Colorado, Denver*	Ulam Spirals and Primal Planes
9:25 - 9:45	Tilsa Aryeni *University of Wyoming*	On the Application of Stable Generalized Finite Element Method for Quasilinear Elliptic Two-Point BVP
9:50 - 10:10	Michael Zowada *University of Colorado, Colorado Springs*	Classification of KPI lumps via Partitions of Integers
10:10 - 10:25	15 Minute Break	
10:25 - 10:45	James Haley *University of Colorado, Denver*	Constructing the history of a wildfire from satellite observations
10:50 - 11:10	Angela Morrison *University of Colorado, Denver*	Earth, Wind & Finite Elements: Approximating Wind Field Interactions with Terrain in Wildfires
11:15 - 11:35	Evan Shapiro *University of Colorado, Denver*	A Tale of Wind and Fire: Improving and Validating a Wildfire Wind Field Model using Finite Elements and Streamlines

Industry Panel & Lunch: 11:45 - 12:30

Zoom ID: 981 3034 9510

Panelists:
- Will Mayfield (*National Center for Atmospheric Research*)
- Sarah Knepper (*Intel*)
- Jennifer Loe (*Sandia National Laboratories*)

Plenary Address: 12:45 - 1:45

Zoom ID: 981 3034 9510

What Can Calculus Tell Us About Life?
Nancy Rodríguez, *University of Colorado, Boulder*

Group Picture: 1:50 - 1:55

Zoom ID: 981 3034 9510

Afternoon Session I: 2:00 - 3:35

Zoom ID: 955 4040 1849

2:00 - 2:20	Andy Gibson *University of Colorado, Colorado Springs*	Koopman Analysis and Control of Nonlinear Bubble Dynamics
2:25 - 2:45	Van Hovenga[1] and Edith Lee[2] [1] *University of Colorado, Colorado Springs* [2] *University of Rochester*	Quasistationary Distribution of the Invasion Model on Bipartite Graphs
2:50 - 3:10	Alex Paradise *University of Colorado, Boulder*	Multilateration of Jaccard Metric Spaces
3:15 - 3:35	Tian Yu Yen *University of Colorado, Denver*	Impacts of Finite Data and Distributional Estimation on Data Consistent Inversion

Afternoon Session II: 2:00 - 3:10

Zoom ID: 922 5393 3279

2:00 - 2:20	Wenjuan Zhang *University of Colorado, Denver*	Get to Know Neural Networks

2:25 - 2:45	Stetson Zirkelbach *University of Colorado, Denver*	Separation-Preserving Transition of Clusterings
2:50 - 3:10	Melissa Butler *University of Wyoming*	Quantifying Uncertainty in the Richards Equation through Brownian Motion

Plenary Speaker

12:45 - 1:45

What Can Calculus Tell Us About Life?

Nancy Rodríguez
Assistant Professor, Department of Applied Mathematics
University of Colorado Boulder

In this talk I will discuss how we can use calculus to gain insight into complex social, ecological, and biological phenomena. We will see how we can use the framework of partial differential equations to encompass many types of phenomena when we are interested in studying global structures, such as the dynamics of a population versus an individual. We will look at applications in urban crime, social outburst of activity, territory formations in ecology. We explore various important mathematical questions from the point of view of the applications and discuss the limitation of our framework.

About the Speaker

Dr. Nancy Rodríguez is an Assistant Professor at CU Boulder Applied Math Department. Her research focuses on nonlinear partial differential equations (PDEs), in particular those with applications to urban crime, segregation, biological aggregation, chemotaxis, and ecology. She is interested in the mathematical modeling and the use of numerical and mathematical analysis to shed light into social, biological and ecological systems. In recognition of her excellence in research and teaching, Professor Rodríguez was awarded a prestigious NSF CAREER award. You may learn more about Dr. Rodríguez at https://www.colorado.edu/amath/nancy-rodriguez.

MORNING SESSION I

LOW DIMENSIONAL EMBEDDABILITY OF MICROBIOME DATA

Evan Gorman
University of Colorado, Boulder

In this talk I will present two tools Microbiologists commonly use to analyze microbial environments. UniFrac: a distance metric on phylogenetic trees and Principal Coordinates Analysis (PCoA): a method to find a Euclidean embedding of a distance matrix. In practice PCoA performs remarkably well on UniFrac data despite it being a Non-Euclidean metric. We explore properties of this embedding and offer potential explanations to its success based on a spectral analysis. This research has been partially funded by the NSF IIS grant No. 1836914.

ASSOCIATION TESTING IN METABOLOMICS DATA WITH AN APPLICATION TO COMPLIMENTARY DIETS IN INFANTS

Nicholas Weaver
University of Colorado, Denver

Advancements in metabolomics data techniques has enhanced the ability to answer questions in human health, particularly the growth development of infants. Diet interventions provide a chance to observe how changes to the gut metabolome impact the growth of infants (e.g. length-for-age Z scores and weight-for-age Z scores). It is believed that these differences in growth trajectory remain throughout life, impacting the likelihood of developing adverse health outcomes such as obesity. A previous study of complimentary protein sources found that infants who consume meat protein were on a steeper length-for-age growth trajectory than infants consuming dairy protein. In this presentation we expand these results by identifying changes in the gut microbiome of infants from the same study to assess a potential mediator for the relationship between complimentary protein source and infant growth. We first discuss necessary data considerations when working with metabolomics data and then share commonly used methods for association testing in longitudinal metabolomics specifically. Linear regression and principal components regression are applied to the infant diet case study. Results show isoleucine, valine, phenylalanine increased and threonine decreased over time among all participants with no statistically significant difference by diet. A principal component highly weighted by branch chain amino acids had a positive association with changes in length-for-age Z scores during the intervention. We conclude with a brief discussion of current issues in metabolomics association testing.

A ROBUST ANALYSIS OF LINEAR METHODS FOR PHENOTYPE PREDICTION

Megan Duff
University of Colorado, Denver

Predicting an individual's phenotypic value from their genetic data is a goal and current research area for the field of genetics. This would not only serve as a public heath tool but could provide researchers with an opportunity to increase power for their analyses by increasing the sample size. The primary difficulty in creating such a model lies in the number of loci that contribute to a disease compared to the sample sizes used in training the model. Several penalized regression, Bayesian regression, and non-linear prediction methods have been developed to account for such limitations, but there is no robust method that performs best in every scenario. This presentation will address the main methods used for phenotype prediction and offer a comparison between the methods among a variety of genetic architecture assumptions for a phenotype, as well as discussing limitations of such methods and possible future areas of research.

INSULIN ACTION MODELS APPLIED TO GLYCEROL AND GLUCOSE YIELD DIFFERENT DYNAMICS

Griffith Hampton
Colorado School of Mines

In response to a glucose challenge, insulin is released from the pancreas and, under healthy conditions: 1) suppresses lipolysis in adipose tissue to decrease glycerol concentrations, reducing fasting metabolic pathways, and 2) regulates glucose concentrations through actions in muscle and liver. Insulin resistance (IR) occurs when more insulin is required to achieve the same metabolic control, and IR may be tissue-specific. Adipose, liver, and muscle tissue exhibit distinct dose-dependent responses to insulin in multi-phase hyperinsulinemic-euglycemic (HE) clamps, but the HE clamp protocol does not address potential differences in the dynamics of an endogenous insulin response. Adipose tissue undergoes an enzymatic adjustment of hormone sensitive lipase while muscle glucose control involves a complex set of intracellular signaling. To investigate the dynamics of insulin acting on adipose tissue, we developed a differential-equations based model that describes the coupled dynamics of glycerol concentrations and insulin action during a six-hour oral glucose tolerance test in female adolescents with obesity and significant IR. The dynamics of insulin acting on muscle and liver were described with the oral minimal model applied to glucose data collected under the same protocol. We found that the peak action of insulin on glycerol occurs earlier and more closely follows changes in plasma insulin concentrations compared to insulin action on glucose. These findings suggest that, in our IR population, the dynamics of insulin action show tissue-specific differences. Specifically, the time scales of insulin action on glycerol and glucose, respectively, indicate that adipose tissue responds more quickly to insulin compared to the delayed insulin response observed in muscle and liver tissue. Improved understanding of the tissue-specific dynamics of insulin action may provide novel insights into IR and the progression of metabolic disease.

THE EFFECT OF A VIRAL LOAD FUNCTION ON EPIDEMIC DYNAMICS

Nicholas W. Landry
University of Colorado, Boulder

In contrast to the common assumption in epidemic models that the rate of infection between individuals is constant, in reality, an individual's viral load determines their infectiousness. We compare the dynamics of an epidemic model with a deterministic, time-dependent infection rate to that of a standard SIR model with a constant, Markovian probability of infection and healing. We predict the epidemic threshold in terms of contact structure for a fully-mixed population, a static contact network, and a temporal network. We compare the trajectories of epidemic spread for a simulation of the SARS-CoV-2 virus on both static and temporal networks using both an SIR model and a time-dependent viral load model. We find that the effects of the viral load and the contact structure on the epidemic threshold are independent of each other and that the total viral *exposure* is the only salient feature of the infection rate function with respect to the epidemic threshold. We find that when we compare the viral load model to the SIR model on either a static or temporal network, the epidemic peak is delayed but more pronounced. In addition, the specific infection rate function has a strong effect on the time dynamics of the epidemic.

HIVE GEOMETRY SHAPES THE RECRUITMENT RATE OF HONEYBEE COLONIES

Subekshya Bidari
University of Colorado, Boulder

Honey bees make decisions regarding foraging and nest-site selection in groups ranging from hundreds to thousands of individuals. To effectively make these decisions bees need to communicate within a spatially distributed group. However, the spatiotemporal dynamics of honey bee communication have been mostly overlooked in models of collective decisions, focusing primarily on mean field models of opinion dynamics. We analyze how the spatial properties of the nest or hive, and the movement of individuals with different *belief states* (uncommitted or committed) therein affect the rate of information transmission using spatially-extended models of collective decision-making within a hive. Honeybees waggle-dance to recruit conspecifics with an intensity that is a threshold nonlinear function of the waggler concentration. Our models range from treating the hive as a chain of discrete patches to a continuous line (long narrow hive). The combination of population-thresholded recruitment and compartmentalized populations generates tradeoffs between rapid information propagation with strong population dispersal and recruitment failures resulting from excessive population diffusion and also creates an effective colony-level signal-detection mechanism whereby recruitment to low quality objectives is blocked.

MORNING SESSION II

ESTIMATING OBSERVATION-CONSISTENT SOLUTIONS USING WEIGHTED EMPIRICAL DISTRIBUTION FUNCTIONS

Kirana Bergstrom
University of Colorado, Denver

We consider a class of stochastic inverse problems involving the characterization of a probability measure on the input parameters of a computational model whose subsequent push-forward matches an observed probability measure on specified quantities of interest associated with model outputs. Such a solution is formally defined by the pullback of the observed probability measure and is referred to as an observation-consistent solution. Previous approaches for approximating observation-consistent solutions relied upon density estimation or set/event approximations. Such approaches are challenging to implement in scenarios where the number of either simulated or observational data are limited. Separate research has tackled the problem of estimating push-forward measures under such scenarios by using weighted empirical distribution functions where the weights are defined as the solution to a constrained quadratic optimization problem. We will demonstrate that a weighted empirical distribution function solving a constrained quadratic optimization problem produces an approximate observation-consistent solution.

APPROXIMATE MAXIMUM LIKELIHOOD ESTIMATORS FOR LINEAR REGRESSION WITH OPERATOR UNCERTAINTY

Richard Clancy
University of Colorado, Boulder

In this talk, we consider regression problems subject to arbitrary noise in the data matrix. This characterization appropriately models many physical phenomena with model uncertainty, particularly for bounded but not necessarily small noise. Although the problem has been studied extensively for ordinary/total least squares, and via models that implicitly or explicitly assume Gaussianity, less attention has been paid to improving estimation for regression problems under general uncertainty in the design matrix. To address difficulties encountered when dealing with distributions of sums of random variables, we rely on a saddle point method to estimate densities and form an approximate log-likelihood to maximize. We show that the proposed method performs favorably against other classical methods.

DELAYED CLASSICAL GRAM-SCHMIDT WITH REORTHOGONALIZATION (DCGS2) - IMPROVED GRAM-SCHMIDT ORTHOGONALIZATION ALGORITHM FOR PARALLEL COMPUTATION

Daniel Bielich
University of Colorado, Denver

The number of global reductions plays an important role in the parallel scalability of Krylov iterative methods. Our focus is on the Arnoldi-QR algorithm for nonsymmetric matrices. In this context, the underlying orthogonalization scheme is “left-looking” and “sees” columns one at a time. Thus, in the standard Arnoldi context, without s-step methods or block methods, at least one global reduction is required per iteration. A stable method for orthogonalizing Krylov vectors during the Arnoldi process is the classical Gram Schmidt algorithm with reorthogonalization (CGS2), requiring three global reductions per step. A new variant of Arnoldi-CGS2 that requires only one reduce has been derived. Numerical stability and strong-scaling results are presented within C and Trilinos. A preliminary attempt to derive a similar algorithm (one reduction per Arnoldi iteration with a robust orthogonalization scheme) was presented by Hernandez et al. 2007 [1] but their method lacks numerically stability; while our new method, after extensive experiments, is much more stable and accurate. Our algorithm can also be implemented in the context of a QR factorization (as opposed to Arnoldi), and we provide results in this context as well.

SEMI-SUPERVISED NON-NEGATIVE MATRIX FACTORIZATION FOR LEARNING TASKS

Lara Kassab
Colorado State University

Supervision-aware dimensionality-reduction techniques have become increasingly important in machine learning; such techniques aim to learn a lower-dimensional representation of the data alongside a supervised learning model. We propose new models for semi-supervised nonnegative matrix factorization as dimensionality reduction techniques for learning tasks. We provide motivation for the models as maximum likelihood estimators given specific distributions of uncertainty. Further, we present training methods for each of the models, and demonstrate their application to classification tasks.

ANALYSIS OF THE ASYMPTOTICS OF THE LOWEST UNIQUE POSITIVE INTEGER GAME

Alyssa Newman
University of Colorado, Denver

In this project we find Nash equilibriums for the lowest unique positive integer game (LUPI) and two variations of it. We are focusing on the asymptotic of the game and how it behaves when there are large number of players, using techniques that can be extracted to other variations. We studied both the standard version of the game and two over variants. One variation is where the player wants to choose the second lowest unique integer and the other variation is where the player wants to find the lowest integer that was chosen some exact, larger than one, number of times rather than being unique. Using numerical techniques we verify the Nash equilibrium for the standard version of the unique lowest integer game and we find several Nash equilibriums for the two other variations of the game. We are currently working on an evolutionary algorithm similar to machine learning to get these same results using a new technique.

APPLYING THE CYCLE CANCELLING ALGORITHM TO THE TRAVELING SALESMAN PROBLEM

Zachary Sorenson
University of Colorado, Denver

The Traveling Salesman Problem (TSP) is an optimization problem that is notoriously difficult to solve. In order to solve a TSP, we want to find the most cost-effective way to visit a set of locations and return to a starting location. An efficient algorithm to find the best solution to the TSP, or the optimal solution, has yet to be found. Instead, when approaching the TSP we can start by finding a solution that simply visits each location, also known as a feasible solution, and not be concerned about the cost of this solution. Finding a feasible solution is almost always easier than finding an optimal solution and can often be found with little to no computational effort. We can then use one of several efficient algorithms that can take a feasible solution and improve it. The purpose of this talk is to discuss an algorithm that is typically used to solve network flows problems called the Cycle Cancelling Algorithm. We will also discuss various algorithms outside of the network flows context that are currently used to improve feasible solutions to the TSP, and how the cycle cancelling algorithm can solve certain problems that these algorithms cannot. For the future, I plan to continue researching the Cycle Cancelling Algorithm and determine if it is an effective way to improve a feasible solution to the TSP, and how effective the algorithm is.

MORNING SESSION III

ULAM SPIRALS AND PRIMAL PLANES

Zane Showalter-Castorena
University of Colorado, Denver

Ulam spirals are a way to visualize prime numbers. This talk will present both 2D and 3D Ulam spirals and hypothesize methods for modifying these spirals to (hopefully) yield more information about prime numbers.

ON THE APPLICATION OF STABLE GENERALIZED FINITE ELEMENT METHOD FOR QUASILINEAR ELLIPTIC TWO-POINT BVP

Tilsa Aryeni
University of Wyoming

We discuss the application of the Generalized Finite Element Method (GFEM) to approximate the solutions of quasilinear elliptic equations with multiple interfaces in one dimensional space. The problem is characterized by spatial discontinuity of the elliptic coefficient that depends on the unknown solution. It is known that unless the partition of the domain matches the discontinuity configuration, accuracy of standard finite element techniques significantly deteriorates and standard refinement of the partition may not suffice. The GFEM is a viable alternative to overcome this predicament. It is based on the construction of certain enrichment functions supplied to the standard space that capture effects of the discontinuity. This approach is called stable (SGFEM) if it maintains an optimal rate of convergence and the conditioning of GFEM is not worse than that of the standard FEM. A convergence analysis is derived and performance of the method is illustrated by several numerical examples.

CLASSIFICATION OF KP I LUMPS VIA PARTITIONS OF INTEGERS

Michael Zowada
University of Colorado, Colorado Springs

We investigate a class of two-dimensional, real, non-singular, rational solutions of the KP I equation. These solutions are termed as the N-lump solutions because they are characterized by N distinct peaks interacting nonlinearly like 2-D solitons. In this talk, I will describe a classification scheme of the KP N-lump solutions based on partition of integers and the related Young diagrams.

CONSTRUCTING THE HISTORY OF A WILDFIRE FROM SATELLITE OBSERVATIONS

James Haley
University of Colorado, Denver

The occurrences of wildfires are increasing both in number and size due to climate change. Effective modeling of fires can help planners and emergency response crews to mitigate the adverse effects experienced by society and the environment. One way to help make more reliable predictions is to initialize the model run using the latest information known about the fire environment. One source of information that is always available comes from satellites observing the ground with infrared cameras. Unfortunately, the locations where satellites believe a fire to be burning are often very sparse and traditional interpolation techniques are inadequate for inferring the state of a fire in all locations of a fire domain. In this talk, we present a new method for reconstructing the history of a fire by tracing the paths most likely to have been taken from an assumed ignition location to all other points in the region subsequently determined by satellites to be burning.

EARTH, WIND & FINITE ELEMENTS: APPROXIMATING WIND FIELD INTERACTIONS WITH TERRAIN IN WILDFIRES

Angela Morrison
University of Colorado, Denver

One major factor in modeling the spread of wildfires is including information about the wind. To gain more information about the wind, one needs to look at details of the terrain since it has a strong effect on wind behavior. Currently the Weather Research Forecasting (WRF) model is what contains information about the wind field, but this is on a rather coarse resolution grid. When a fire model, SFIRE, is coupled with WRF, the wildfires are modeled on a finer resolution grid which takes into account more details about the terrain. The issue lies in that the SFIRE model needs to be able to use the wind information from the coarser scale WRF grid and apply it to the finer grid. Currently the wind field from WRF is interpolated to the SFIRE grid, but this alone does not give results that would be good at approximating wind behaviors such as acceleration on top of hills or the change in direction on the sides. This project is to replace the interpolation by the solution of a reduced flow model, which computes flow with zero mass divergence on the fine grid over the fine-scale terrain. To minimize the difference between the known wind field in the WRF model and the fine-scale wind field of the SFIRE model, a finite element method is applied which results in a down-scaling of the wind field information. This presentation will briefly cover the basics of the finite element method, its use in the wind down-scaling process, and how it is applied when solving for the wind-field over the fine-terrain following grid.

A TALE OF WIND AND FIRE: IMPROVING AND VALIDATING A WILDFIRE WIND FIELD MODEL USING FINITE ELEMENTS AND STREAMLINES

Evan Shapiro
University of Colorado, Denver

In this presentation we discuss developments in the WRF-SFIRE code, a wildfire spread model that which is coupled to the Weather Research and Forecasting Model (WRF) atmospheric model. The fire model uses a fine scale numerical grid of the wind field from coarse scale wind data (down scaling) by interpolating input wind data from WRF. An accurate wind field is key for accurately predicting how a wildfire will spread. We are particularly interested in interpolating the wind field close to the earth's surface, where fine scale terrain features affect the wind field. The wind field at fine scales can be approximated as a divergence free field, and with our new software we seek to minimize the amount of divergence that interpolation introduces to the wind field. Mathematically speaking we are minimizing the difference between an unknown divergence free wind field and a known windfield with divergence, with respect to the L 2 norm. This minimization problem can be solved using the finite element method. In this talk I discuss how the stretched grid is constructed numerically. I also discuss code validation via plotting of the streamlines of the resulting wind field after a full simulation run with a new streamlines function. This validation allows us to detect anomalies in the resulting wind field, and potential issues with seed data, and cannot be performed with Matlab's included streamline function due to the way in which the numerical grid is constructed.

AFTERNOON SESSION I

KOOPMAN ANALYSIS AND CONTROL OF NONLINEAR BUBBLE DYNAMICS

Andy Gibson
University of Colorado, Colorado Springs

Volume and shape oscillations of gas bubbles in liquids form a central area of study in multiphase fluid mechanics, with important applications to intravenous drug delivery, contrast-enhanced ultrasound imaging, and cavitation-induced flow instabilities and erosion in turbomachinery. In this study, we use emerging tools from Koopman operator theory to analyze the Rayleigh-Plesset equation governing spherical bubble oscillations. Koopman theory is a framework that provides a globally linear representation of even strongly nonlinear dynamical systems. Such a Koopman embedding allows for simple future state prediction and the application of classical control techniques, including optimal control. Through combination with data-driven and machine learning methods, coherent spatio-temporal structures can be extracted and a controller can be trained on either numerically simulated data or experimentally obtained time series. Here we use algorithms called Hankel-DMD (Dynamic Mode Decomposition) and SINDy (Sparse Identification of Nonlinear Dynamics) to extract eigenfunctions of the Koopman operator for the RPE. These nonlinear functions then provide a basis, analogous to Fourier modes, for a linear embedding of the nonlinear dynamics. Fundamental frequencies and harmonics emerge naturally. Finally, these eigenfunctions are used as coordinates to develop an LQR controller for the nonlinear system. Authors: Andrew J. Gibson, Dr. Michael L. Calvisi, Dr. Xin (Cindy) Yee Association: University of Colorado Colorado Springs, Department of Mechanical and Aerospace Engineering

QUASISTATIONARY DISTRIBUTION OF THE INVASION MODEL ON BIPARTITE GRAPHS

Van Hovenga[1] and Edith Lee[2]
[1] ***University of Colorado, Colorado Springs***
[2] ***University of Rochester***

The (discrete-time) invasion model on a finite graph is a Markov Chain where a state is some coloring of vertices of the graph (the different colors represent different opinions). At each time step, the invasion model goes to the next state by randomly selecting a vertex and forcing its color (opinion) to one of its neighbors which is also chosen by random. Therefore, when all the vertices are the same color, referred to as reaching consensus, the model will remain in that state. If the graph is connected, consensus is reached almost surely. However, conditioning that consensus has not been reached, the probabilities will approach some distribution, known as the quasistationary distribution (QSD) for the model. The goal of this project is to analyze these distributions for complete bipartite graphs. We explore the rate of convergence to the QSD and offer a technique to find the closed form of this distribution using coalescing random walks.

MULTILATERATION OF JACCARD METRIC SPACES

Alex Paradise
University of Colorado, Boulder

Without question, social media websites such as Twitter have become an integral part of our daily lives. Unfortunately, the heavy use of these platforms has also led to a rise in misinformation. In this talk, I will show how the Jaccard distance could be used as a tool to represent tweets as numerical vectors, hoping to use this to, for example, distinguish bots' tweets from humans. We address this by abstracting words as elements of a set X and identifying tweets as subsets of X. The Jaccard distance is then used to multilaterate the power-set of X. Multilateration is an abstraction of the trilateration of the plane to a metric space: it consists of finding landmarks (points) in that space so that any other point is uniquely determined by its distance to those landmarks. A lower bound for the number of landmarks needed will be provided, and other results that may aid in identifying the smallest number of them required to multilaterate the power-set of X. This research has been partially funded by the NSF IIS grant No. 1836914.

IMPACTS OF FINITE DATA AND DISTRIBUTIONAL ESTIMATION ON DATA CONSISTENT INVERSION

Tian Yu Yen
University of Colorado, Denver

Data consistent inversion is a method to solve inverse problems that allows researchers to determine a probability distribution on input parameters whose push-forward is consistent with the distribution of observable data. Specifically, given a model map from input parameters to output observations, the resulting data consistent update can be defined as an initial probability distribution describing the input parameters weighted by the ratio of an observed data distribution to the predicted data distribution given the initial distribution of parameters (push-forward of initial). Previous research into data consistent inversion has proven that the data consistent update is stable with respect to perturbations in the observed distribution in the total variation metric. Thus, if an observed data distribution can be specified or can be well-approximated, so too can the data consistent update. The current research focuses on different techniques for obtaining a density estimate from finite observations of data and the implications for the error of the data consistent update in the parameter space. Theoretical and numerical convergence rates are shown and compared for various techniques including kernel density estimates, parametric-mixture models, and the non-parametric Dirichlet process.

AFTERNOON SESSION II

GET TO KNOW NEURAL NETWORKS

Wenjuan Zhang
University of Colorado, Denver

In this talk, I will talk about the basics of neural networks. Examples, figures, and tables will then be used to help build an intuition of how to choose the number of hidden neurons, how to choose the number of epochs, etc.

SEPARATION-PRESERVING TRANSITION OF CLUSTERINGS

Stetson Zirkelbach
University of Colorado, Denver

The separability of clusters is one of the most desired properties in clustering. There is a wide range of settings in which different clusterings of the same data set appear. A common application lies in the need for a transition of one separable clustering into another one. This transition should be a sequence of simple, natural steps that upholds separability of the clusters throughout.

In this presentation, we discus an algorithm for such a transition. This includes a brief overview of intimate connection of separability and linear programming over bounded-shape partition and transportation polytopes that the algorithm utilizes. Then a discussion of the algorithm itself in terms of a sample problem through a combination of two walks: an edge walk between two so-called radial clusterings in a transportation polytope, computed through an adaptation of classical tools of sensitivity analysis and parametric programming; and a walk from a separable clustering to a corresponding radial clustering, computed through a tailored, iterative routine updating cluster sizes and re-optimizing the cluster assignment of items. Lastly, there will be a talk on where this work is going and the features of this work that we are exploring further like the warm-start set up that is used during the walks and the transition between polytopes.

QUANTIFYING UNCERTAINTY IN RICHARDS EQUATION THROUGH BROWNIAN MOTION

Melissa Butler
University of Wyoming

Richards Equation models water flow in an unsaturated porous medium and is used in hydrology, agriculture, waste management, and other fields. The one-dimension mixed-form of Richards Equation is one of the most complex flow models and has limited closed form solutions. Numerical solutions can be obtained by linear discretization of the spatial and temporal variables using a finite volume method. A quantity of interest being studied is the total water content at a given time. By introducing Brownian Motion to the Neumann boundary condition and analyzing the total water content we can see the effect of the uncertainty of on the surface. From this, a mean and standard deviation can be built and the respective distribution made available.