APPM 5720: Bayesian Statistics Exam I, Take-Home Part, Spring 2018

(Due, Monday March 12th)

This is an exam. Please do not discuss it with others. You may use any books or websites for help as long as you ultimately give a fully explained solution. (i.e. Don't write things like "According to Wikipedia, the limiting distribution is...") Have fun.

1. Let X be a binomial random variable with parameters n and θ. Choose a prior on θ such that the marginal distribution of X is uniform over $\{0, 1, 2, \ldots, n\}$.

2. In the review problems for the in-class part of Exam I, we found a Jeffreys prior that was improper. We can work with improper priors as long as the posterior distribution is proper.
   (a) Give a heuristic reason why the Jeffreys prior "usually" results in a proper posterior.
   (b) Give an example where the Jeffreys prior does **not** result in a proper posterior!

3. Recall that $A \sim W_p(n, V)$ with $n \geq p$ means that the $p \times p$ positive definite matrix A has a Wishart distribution with n degrees of freedom and variance-covariance parameter matrix V. Recall also that the pdf for A is given by

$$f(A) \propto |A|^{(n-p-1)/2} \exp\left[-\frac{1}{2}tr(V^{-1}A)\right].$$

   Suppose now that $A^{-1} \sim W_p(n, V)$.

   One can show (but you don't have to) that A follows an "inverse Wishart" distribution, with pdf

$$f(A) \propto |A|^{-(n+p+1)/2} \exp\left[-\frac{1}{2}tr(VA^{-1})\right].$$

   In this case we write $A \sim IW_p(n, V)$ or $A \sim W_p^{-1}(n, V)$.

   Suppose that $A \sim IW_p(n, V)$. Partition A into

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

   and V into

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$$

   where A_{11} and V_{11} are $q \times q$ matrices for some fixed $q \leq p$.

   Find the distribution of A_{11}.