

Attraction, merger, reflection, and annihilation in magnetic droplet soliton scattering

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The interaction behaviors of solitons are defining characteristics of these nonlinear, coherent structures. Due to recent experimental observations, thin ferromagnetic films offer a promising medium in which to study the scattering properties of two-dimensional magnetic droplet solitons, particle-like, precessing dipoles. Here, a rich set of two-droplet interaction behaviors are classified through micromagnetic simulations. Repulsive and attractive interaction dynamics are generically determined by the relative phase and speeds of the two droplets and can be classified into four types: (1) merger into a breather bound state, (2) counterpropagation trapped along the axis of symmetry, (3) reflection, and (4) violent droplet annihilation into spin wave radiation and a breather. Utilizing a nonlinear method of images, it is demonstrated that these dynamics describe repulsive/attractive scattering of a single droplet off of a magnetic boundary with pinned/free spin boundary conditions, respectively. These results explain the mechanism by which propagating and stationary droplets can be stabilized in a confined ferromagnet.

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Solitary waves or solitons are particle-like wave packets that arise in a wide range of physical contexts from a balance between dispersive spreading and nonlinear focusing. One of the key phenomena that differentiates nonlinear coherent structures such as solitons from their linear counterparts is what happens when such structures interact. Soliton solutions of equations with very special mathematical structure (integrability) have been shown to interact elastically [1] and can be attractive or repulsive [2]. In more general systems, soliton interactions can be significantly more complex, exhibiting fusion, fission, annihilation, or spiraling [3,4]. A relative phase between the solitons plays a dominant role in determining the resulting interaction behaviors. An additional interaction feature, 90° scattering, has been predicted for two-dimensional (2D) magnetic solitons [5,6] and solitons in field theories [7,8]. The recent experimental observation of a magnetic droplet soliton in a spatially extended film [9] provides the impetus for our deeper study of magnetic soliton interactions. Here, we show that the interaction of a pair of 2D magnetic droplet solitons (from here on in, droplets) exhibits rich behavior, principally dependent on the droplets' relative phase.

Previous studies of soliton interaction in 2D ferromagnetic materials have concentrated primarily on vortices, topological structures that exhibit restricted dynamics [10]. Unless the ferromagnet is confined [11], conservation of overall topological charge pins the magnetic “center of mass” in place, e.g., a single vortex core, limiting motion to rotating collections [5,12] or linear motion of vortex pairs with net zero topological charge. Perpendicular scattering of two interacting vortex pairs has been theoretically demonstrated [6]. It appears that 90° scattering has a more universal character [8], not requiring a topological charge, and previous numerical studies have indeed shown perpendicular scattering even for approximate

nontopological solitons [5]. Loosening topological restrictions and the fact that droplets, due to their precessional nature, possess an extra degree of freedom (phase) opens up many fascinating modes of interaction.

In this Rapid Communication, we classify head-on and angled droplet interactions in terms of the droplets' relative phase and momenta via micromagnetic simulations. Sufficiently in-phase droplets experience an attractive interaction that results in either merger into a new breathing bound state for low speeds, or a scattering event transferring droplet motion to the axis of symmetry. Out-of-phase droplets experience a repelling interaction that results in a scattering event obeying the law of reflection. Via symmetry, these results show that a ferromagnetic boundary with free (pinned) spins attracts (repels) a single droplet. In particular, this provides an explanation for the existence of “edge droplets” theoretically predicted for a spin torque driven, confined ferromagnet with a free spin boundary [13]. Finally, at an intermediate relative phase, the colliding droplets exhibit an “explosion” into spin waves and the spontaneous formation of a single, breathing droplet. This annihilation behavior mimics particle colliders in which high-energy particles are smashed into byproducts.

The model we consider is the Landau-Lifshitz torque equation with perpendicular anisotropy,

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times [\nabla^2 \mathbf{m} + (m_z + h_0)\mathbf{z}], \quad (1)$$

where $\lim_{|\mathbf{x}| \rightarrow \infty} \mathbf{m} = \mathbf{z}$. Equation (1) is an ultrathin-film 2D reduction of the full Landau-Lifshitz equation with long-range magnetostatic effects [14]. The magnetization vector is normalized to unit length, spatial lengths are in units of $L_{\text{ex}}/\sqrt{Q-1}$, times are in units of $[|\gamma|\mu_0 M_s(Q-1)]^{-1}$, and the perpendicular magnetic field magnitude $h_0 > 0$ is scaled by $M_s(Q-1)$, where L_{ex} is the exchange length, γ is the gyromagnetic ratio, μ_0 is the free space permeability, M_s is the saturation magnetization, $Q = 2K_u/(\mu_0 M_s^2)$ is the dimensionless quality factor, and K_u is the crystalline perpen-

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dicular anisotropy constant. Here it is assumed that $Q > 1$ or equivalently that the perpendicular anisotropy is sufficiently strong that it overcomes the effective planar anisotropy due to the magnetostatic field. This assumption is not an excessive restriction as ferromagnets with this property such as CoFeB or Co/Ni multilayers are currently in use (cf. [9]). The energy $\mathcal{E} = \frac{1}{2} \int [|\nabla \mathbf{m}|^2 + 1 - m_z^2 + h_0(1 - m_z)] dx$ is conserved by solutions of (1). The magnetic field induces a positive shift of precession frequency. By entering the rotating frame, we take $h_0 = 0$ without loss of generality.

Droplet solutions of Eq. (1) are parameterized by six distinct quantities: initial position \mathbf{x}^0 , initial central phase ϕ^0 , propagation velocity \mathbf{V} , and rest precession frequency ω [14]. A previous droplet interaction study was limited to accurately computed stationary (radially symmetric) droplets [5]. These solutions were artificially deformed to induce propagation with a fixed but not prescribed speed and were accompanied by radiation. Only in-phase, head-on, approximate droplet interactions were considered. In this Rapid Communication, we leverage translation, rotation, and phase invariance of Eq. (1) in combination with a very accurate database of precomputed propagating droplets [14] in order to explore a wide range of two-droplet initial conditions, each droplet parameterized by $(\mathbf{x}_i^0, \phi_i^0, \mathbf{V}_i, \omega_i)$, $i = 1, 2$. The angle of interaction ψ is the angle between \mathbf{V}_1 and \mathbf{V}_2 . See [15] for micromagnetic details.

All the interactions described here depend principally on the relative phase $\Delta = \phi_1^0 - \phi_2^0$ of the two initial droplets. We find that the interaction can be broadly classified as attractive or repulsive with maximal attraction when $\Delta = 0$ varying to maximal repulsion when $|\Delta| = \pi$, much as is the case for optical solitons [3], demonstrating the universality of this behavior. There is a critical, crossover phase $\Delta_{cr} > 0$ that divides the attractive and repulsive regimes. Within this general classification, there are four modes of interaction depending on Δ and $\mathbf{V}_{1,2}$. Figure 1(a) (small V_i , $|\Delta| < \Delta_{cr}$):

merger of two droplets into a bound state whose perimeter is modulated (“breathes”) with twice the precessional frequency. Figure 1(b) (V_i large enough, $|\Delta| < \Delta_{cr}$): merger followed by counterpropagating droplets trapped along the axis of symmetry defined by $\mathbf{V}_1 + \mathbf{V}_2$. Figure 1(c) (any V_i , $\Delta_{cr} < |\Delta| \leq \pi$): reflection off the symmetry axis. Figures 1(d)–1(g) (V_i large enough, $|\Delta| \approx \Delta_{cr}$): droplet merger and annihilation into spin waves and a single propagating breather soliton. Animations of all cases are available [16]. We now describe each interaction category.

First, we consider the interaction of two stationary droplets, $\mathbf{V}_i = 0$, initially situated so they weakly interact (10 units apart). The initial droplets have the same frequency $\omega_1 = \omega_2 = \omega$, but varying relative phase. For $\Delta_{cr} < |\Delta| \leq \pi$, the droplets slowly propagate away from one another, exhibiting weak repulsion. For $|\Delta| < \Delta_{cr}$, the attraction interaction results in merger and then perpendicular scattering. Lacking sufficient momentum to overcome the attraction, this merge-scatter process occurs many times, each with a small loss of energy in the form of radiating spin waves until the structure stabilizes into a breather state. This two-droplet bound state exhibits two frequencies: a precessional and a breathing frequency, twice that of the precessional, at which the shape of the new structure oscillates. We have checked the numerically stable evolution of the breather in Fig. 1(a) by evolving it for 1400 time units. For initial droplet frequencies $\omega = 0.4$, the resulting new structure has precession frequency 0.3 and exhibits a deformation of shape as in the quarter-period oscillation between the two configurations depicted in Fig. 1(a). This new solitary wave is distinctly different from the stationary droplet and what was observed in the previous numerical study [5] where, for $\Delta = 0$, the two droplets were observed to merge-scatter, radiate spin waves, and settle to a new, pure droplet with a single frequency. This merging behavior is similar to soliton fusion observed in optics [17].

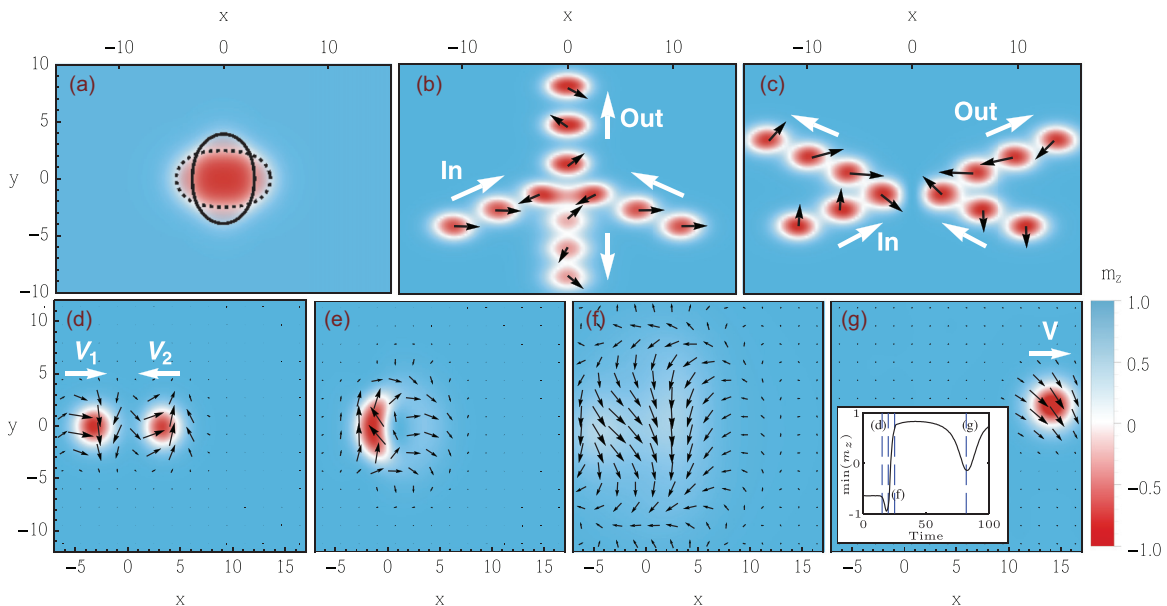


FIG. 1. (Color) Droplet interactions. (a) Breathing droplet at two times. (b) In-phase merger and counterpropagation. (c) Out-of-phase reflection. (d)–(g) Droplet merger (d), annihilation to magnons (e), spontaneous breather formation (f), and propagation (g). Inset to (g): Spatial minimum of m_z as a function of time for (d)–(g); vertical dashed lines denote times in (d)–(g).

The next class of interactions we investigate are propagating droplets with equal frequency $\omega_1 = \omega_2 = \omega$, equal speed $V_1 = V_2 = V$, and velocities reflected, $V_{1,x} = -V_{2,x}$, about the y axis so that \mathbf{y} represents the axis of symmetry. When the angle of interaction $|\Delta| = \pi$, the collision is head-on. The attractive interaction $|\Delta| < \Delta_{cr}$ leads to merger and “trapped” scattering along the y axis as in Fig. 1(c). For the symmetric case when $V_{i,y} = 0$, the scattering is 90° . For the repelling interaction, $\Delta_{cr} < |\Delta| \leq \pi$, the droplets reflect at an angle equal to the angle of incidence $\psi/2$, as in Fig. 1(b). Both Figs. 1(b), 1(c) have $\omega = 0.4$, $\psi = 2\pi/3$, $V = 0.6$, and successive plotted droplets are $t = 10$ units apart. As $|\Delta|$ approaches Δ_{cr} , the two droplets collide with one preferentially absorbing the other, transferring a significant portion of their energy into spin waves followed by the spontaneous formation of a breather state as shown in the head-on collision of Figs. 1(d)–1(g) with $\Delta = \Delta_{cr} = 92^\circ$, $\omega = 0.4$, $V = 0.6$, and $t \in (30, 40, 80, 164)$. The asymmetry in the interaction of Figs. 1(e)–1(g) is due to the choice $0 < \Delta < \pi$. A change in the sign of Δ reverses the asymmetry. Figure 1(g) (inset) demonstrates a steep depletion of the excitation amplitude $1 - m_z$ during the loss of energy to spin waves and an amplitude coalescence associated with the formation of the breather. Annihilation therefore represents the crossover from attractive to repulsive scattering where the incommensurate phases of the colliding droplets cannot be resolved at high kinetic energies, resulting in the explosive release of spin waves accompanied by breather bound state formation.

Previous observations of soliton annihilation in optics were of a very different type [18] where the simultaneous collision of three solitons could result in annihilation of only one of them. Here we see interaction behavior reminiscent of high-energy particles in a collider. The by-products of droplet collision are a shower of magnons (spin waves) and a localized breather. Because a single droplet can be interpreted as a bound state of magnon quasiparticles [19], the annihilation interaction results in the irretrievable loss of energy to fundamental constituents and a conglomerate state.

Now we investigate the interaction classification as both the initial frequency $\omega = \omega_1 = \omega_2$ and velocities $\mathbf{V}_1 = (V, 0) = -\mathbf{V}_2$ are varied for the head-on collision configuration. Figure 2(a) depicts the variation in the attractive to repulsive crossover parameter Δ_{cr} . Generally, for any initial speed V , Δ_{cr} decreases with increasing ω showing that the repulsive interaction is favored for smaller amplitude droplets (increasing ω , V lead to a decrease in droplet amplitude [14]). Colliding droplets with $V \in \{0.3, 0.6\}$ exhibit approximately the same crossover, in contrast to $V = 0$, which is downshifted by about 30° – 40° . We deduce that propagating droplets favor attractive scattering across a wider range of phases than initially stationary droplets. Moving droplets exhibit an underlying spin-wave-type structure with wave number $k = V/2$ that is associated with nonzero local topological density [5,14], whereas stationary droplets have a uniform phase and zero topological density [19]. We never observed stationary droplets to annihilate so the difference in Δ_{cr} for moving and stationary droplets and the existence of annihilation may be attributable to the complexity introduced by nonzero k and topological density associated with propagating droplets.

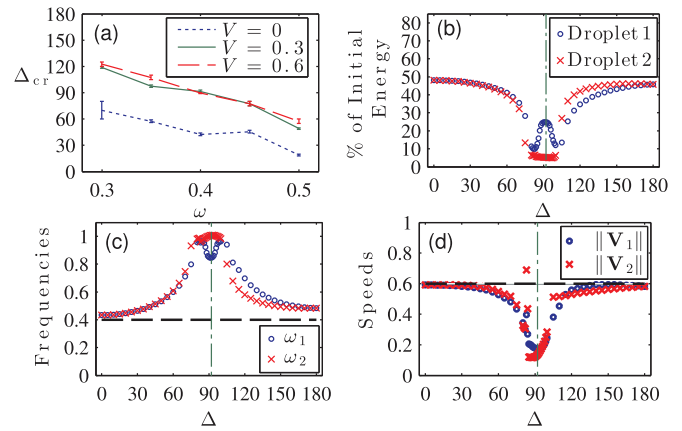


FIG. 2. (Color online) Head-on collision properties. (a) Crossover phase for varying initial ω , V . (b)–(d) Postcollision properties for initial $\omega = 0.4$, $V = 0.6$. Scattered droplet energy (b), frequency (c), and speed (d).

When $0 < |\Delta| < \Delta_{cr}$, the result of attractive interaction is droplets of different speeds and frequencies relative to their initial values. The post-interaction droplet properties for head-on collisions with varying Δ are shown in Figs. 2(b)–2(d). Generically, in-phase and out-of-phase interactions exhibit symmetric behavior with the least energy loss to radiation. For most Δ below Δ_{cr} , the post droplet frequencies and speeds are roughly symmetric. Near and above the crossover Δ_{cr} , however, there is significant asymmetric energy loss due to increased spin wave radiation and energy exchange between the droplets with their frequencies and speeds approaching the linear spin wave band $\omega = 1 - V^2/4$ [14]. As Δ is increased, Fig. 2(b) shows the energy retained in the post-scattered droplets decreasing until it reaches a minimum. Above this value of Δ we begin to observe the annihilation interaction. The local extremum [at 92° for the parameters in Fig. 2(b)] serves as the definition of Δ_{cr} . When $\Delta_{cr} < \Delta < \pi$, the post-interaction droplet with greater energy is reflected to the right, the roles reversed if the sign of Δ is changed.

For in-phase droplets not propagating head-on, we observe droplet scattering along the direction $\mathbf{V}_1 + \mathbf{V}_2$. The asymmetry in energy transfer post interaction is accentuated in Fig. 1(b). This asymmetry is due to the conserved positive momentum in the direction $\mathbf{V}_1 + \mathbf{V}_2$, favoring larger droplets. For small ψ , the collision results in approximately a single droplet. This behavior varies in a continuous fashion, limiting to the case when $\psi = \pi$ for 90° scattering of droplets with equal size and frequency.

The model (1) neglects several important physical effects. For example, relaxation processes (damping) in ferromagnets are typically weak but play an important role in experiments and soliton dynamics [20]. Long-range magnetostatics affects any ferromagnet with finite thickness. We numerically investigate the impact on droplet interactions due to Landau-Lifshitz damping with damping parameter $\alpha = 0.01$ and a 2D, thickness-dependent correction to the magnetostatic field [21,22] with thickness parameter $\delta = 0.5$. We did not observe a significant qualitative change in the resulting numerical experiments. As observed previously, the effect of damping is to cause droplets to accelerate while

the frequency increases [20,23]. Magnetostatics result in a negative frequency shift of the droplet [22]. We find that the quantitative locations of the breathing, counterpropagation, reflection, and annihilation regions are changed under these perturbations. Nevertheless, we still observe all four phenomena over sufficiently short time scales so that damping has not completely relaxed the magnet to equilibrium.

Finally, we investigate droplet collisions with different frequencies and velocities. Droplet pairings are chosen so that their frequencies and velocities differed but their momenta did not. This has a significant impact on the post-collision frequencies and velocities. The four interaction categories are all observed. Common interaction behaviors are reflection or annihilation resulting in a breather propagating in the direction of overall momentum.

While the case of two initial droplets with the same speed and frequency may seem restrictive, it is highly relevant in applications. Real ferromagnets can exhibit boundaries with either pinned ($\mathbf{m} = \mathbf{z}$) or free ($\partial\mathbf{m}/\partial\mathbf{n} = 0$, \mathbf{n} a boundary normal) spins. We can utilize symmetries of the droplet solution [14] and of Eq. (1) in order to implement a method of images whereby we reflect an initial droplet about the y axis, taking $V_x \rightarrow -V_x$. The choice of two in-phase droplets corresponds to an even reflection and a free spin boundary condition along the y axis. The choice of two out-of-phase droplets corresponds to an odd reflection of (m_x, m_y) and an even reflection of m_z leading to the pinned spin boundary condition $m_z = 1$. Thus, the entire discussion of interacting droplets with $\Delta = 0$ or $\Delta = \pi$ [e.g., Figs. 1(a)–1(c)] translates to droplet scattering off a boundary. This is directly illustrated in Fig. 3 where an in-phase, head-on collision with $(\omega, V) = (0.4, 0.6)$ results in 90° scattering and the generation of two edge droplets counterpropagating along the y axis. The free spin boundary therefore attracts droplets. This has been observed in micromagnetic simulations of ferromagnetic nanowires with a spin torque nanocontact (NC) [13]. The attractive force of

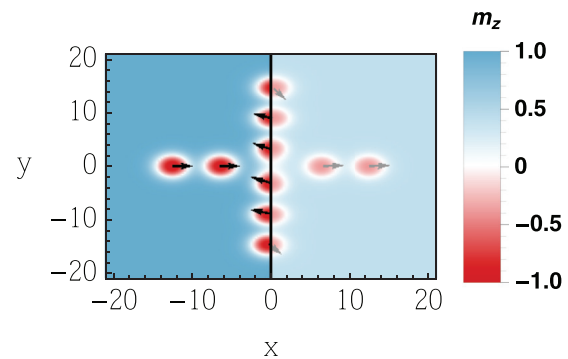


FIG. 3. (Color online) Method of images depicting head-on collision of droplet with a free spin boundary (vertical line). Two counterpropagating edge droplets are created.

the free spin boundary overcomes the restoring force of the NC [22] resulting in an edge droplet for sufficiently narrow wires. Since out-of-phase droplets repel one another, the pinned spin boundary repels droplets. This suggests a way to create a droplet waveguide in a nanowire. If both edges of the nanowire are pinned with vertical magnetization, the droplet is repelled from the boundary. This observation coupled with the ability to accelerate droplets with magnetic field gradients [23] suggests a practical method to stably propagate droplets in patterned media.

In summary, we have classified the interactions of two magnetic droplet solitons into four types depending on their relative phase and speed, observing a new nontopological structure, the droplet breather, and demonstrating attractive, repulsive, and annihilation behaviors.

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- [15] Micromagnetics involve a Fourier pseudospectral method in space and an adaptive, Runge-Kutta time stepper as in [14] with strong magnetic damping applied only near the boundary edges to simulate an absorbing boundary condition. The droplet center of mass is computed from $\bar{\mathbf{x}} = \int \mathbf{x}(1 - m_z)d\mathbf{x} / \int (1 - m_z)d\mathbf{x}$ with the trapezoidal rule and the central phase is extracted via Fourier interpolation at $\bar{\mathbf{x}}$. Velocity, frequency are computed by numerical differentiation. We use a 130×130 domain, 832^2 points, initial droplets 50 units from the boundary, and 10–25 units apart. We monitored the conserved quantities for Eq. (1) (energy, total spin, and momentum) to remain within 2% of the initial value. Numerical parameters validated by convergence to absolute error 10^{-6} of a head-on collision with $\omega = 0.4$, $V = 0.6$, $\Delta = 100^\circ$, over $t = 100$.

- [16] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.89.180409> for animations of Figs. 1(a)–1(g).
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