

Preface

The topics of *Complex Variables* and *Analytic Functions* are of fundamental importance throughout applied mathematics and engineering. Their theorems and formulas also simplify many results from Calculus and for functions of real variables. Not surprisingly, there is already a wide choice available of introductory text books, raising the question why anyone would want to see yet another one. Our main answer lies in the evolution that has occurred in other fields. For at least the last half century or so, it has been unthinkable to use introductory text books for Calculus that do not graphically illustrate the basic elementary functions, such as $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, $f(x) = \log x$, etc. When we first taught a course on Complex Variables and Analytic Functions, we became puzzled about why complex variables texts should not also visually build on the real cases, familiar to all students, and then liberally illustrate how these same functions extend away from the real axis. While formulas alone for some students might provide a feasible alternative to visual impressions, we are here aiming the text for students that find the latter to be helpful for gaining at least their initial intuitive feeling for the subject.

Another issue where text books differ is the order in which topics are covered. One strategy is to make sure each step follows rigorously from previous steps. While that can have some appeal, it might not at all be the best order for building up conceptual understanding as effectively as possible. We will instead focus on the latter, which might at times require issues to be re-visited once additional tools have fallen in place. In either case, the end knowledge will be similar, but it is our belief that the latter approach is the one best suited for an introductory text. A case in point concerns singularities, where we here bring these up early on, before having developed all the tools of contour integration that are needed for a certain (Laurent-expansion based) singularity characterization. To *arrive* at that stage, some earlier insights into singularities are very helpful.

We have in this book never sacrificed correctness for simplicity. However, there inevitably arises situations where it becomes unavoidable to choose a compromise path between the extremes of strictest mathematical rigor (some might call nitpicking) and quite heuristic arguments. Believing that the former is better suited for finalizing proofs than for gaining insights and for solving problems (and can be acquired when needed from sources with that as focus), our preference here has generally been somewhat in favor of the latter.

The emphasis that is given here to different topics also differs somewhat from many other texts. For example, we view analytic continuation as fundamental to a good understanding of the nature of analytic functions. Instead of just listing one approach for this task (the circle-chain theorem, which incidentally is quite impractical for computations), we have included numerous approaches. We also discuss and illustrate multi-valued functions and their Riemann sheets more extensively than what is customary in an introductory text, while we somewhat de-emphasize for example conformal mappings.

The present material originated from lecture notes first developed by Bengt Fornberg in 2006, and then used jointly by us in 2008 (at University of Colorado, Boulder). After that, they were mostly put aside until Cécile Piret revitalized them when resuming teaching the topic in 2015, at Michigan Technological University.

As noted above, a main purpose of extending from the real axis to the complex plane is to greatly simplify a vast number of tasks in applied mathematics, engineering, etc. The eminent mathematician Jacques Hadamard expressed this succinctly (paraphrasing an earlier statement from 1900 by Paul Painlevé):

“The shortest path between two truths in the real domain passes through the complex domain”

However, complex numbers go far beyond being just a mathematical trick that, once having done its magic, ought to quickly and gracefully disappear. It is our hope that this book will make its readers not only appreciate their utility, but also come to regard them as equally ‘natural’ as, say, negative integers.

Acknowledgments: