University of Colorado at Boulder
Department of Applied Mathematics

Comprehensive Examination

Tuesday, April 30, 2013
11:00-12:00 p.m.
Grandview Conference Room at 1320 Grandview Ave

Presenter

Christopher Leibs

Title

Multigrid Preconditioning Methods for an Implicit Jacobian-Free Newton-Krylov Two-Fluid Kinetic Plasma Solver

Plasma is a state of matter in which a collection of atoms have dissociated into their constituent ions and electrons. The availability of this free charge allows the plasma to interact with electromagnetic fields. The fields dictate the motion of the particles and, simultaneously, the motion of the particles alter the fields. This strongly coupled, self-consistent interaction supports a large variety of waves (both slow and fast) that prove to be difficult when attempting simulation. Many computational techniques have been used in plasma simulation, but the two most widely in use are particle-in-cell (PIC) and magnetohydrodynamics (MHD). Recently, a fully implicit electrostatic 1D charge- and energy- conserving PIC algorithm was proposed and implemented [1]. Central to the algorithm is a Jacobian-Free Newton-Krylov (JFNK) method. Internally, each residual calculation requires a call to the costly particle integration scheme. Thus, a well designed preconditioner becomes crucial.

The use of physics based preconditioning (PBP), in conjunction with JFNK, has proved fruitful in applications where disparate time scales are present [2]. Stiff hyperbolic PDE systems are effectively transformed into a scalar parabolic PDE that targets the fast wave behavior. In this way, implicit time-integration schemes are capable of stepping over the stiff time scale while maintaining a reasonable number of Krylov iterations. Moreover, the scalar parabolic PDE is amenable to fast multigrid methods, adding little to the overall computational cost.

The First-Order System Least-Squares (FOSLS) methodology has also been successfully applied to a large variety of problems [3]. First-order systems of PDEs are recast as the minimization of a functional. The resulting weak formulation admits a bilinear form that, if H^1 elliptic, can be used with standard finite element spaces to generate a discrete system amenable to multigrid methods. Recently, the FOSLS approach has been successfully applied to incompressible, resistive MHD [4].

The aim of this research is to use a two-fluid MHD model to build an effective preconditioner for the fully implicit PIC simulation. To be effective, the preconditioner must target fast plasma waves while remaining tractable for multigrid methods. Ideas from both PBP and FOSLS will be explored in the design of the preconditioner.

References

[1] G. Chen, L. Chacón, D. C. Barnes An energy- and charge-conserving, implicit, electrostatic particle-in-cell algorithm. J. Comput. Phys. 2011

[2] D. A. Knoll and D. E. Keyes. Jacobian-free Newton-Krylov methods: a survey of approaches and applications. J. Comput. Phys. 2004

[3] Z. Cai, R. Lazarov, T. A. Manteuffel and S. F. McCormick. First-Order System Least Squares for Second-Order Partial Differential Equations: Part I. SIAM Journal on Numerical Analysis. 1994

[4] J. H. Adler, T. A. Manteuffel, S. F. McCormick, and J. W. Ruge. First-Order System Least Squares for Incompressible Resistive Magnetohydrodynamics. SIAM J. Sci. Comput. 2010