University of Colorado at Boulder
Department of Applied Mathematics

Comprehensive Examination

Wednesday, December 11, 2013
3:00-4:00 p.m.
ECOT 831

Presenter

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Title

Qualitative Analysis of Some Nonlinear PDE systems

The Hardy-Littlewood-Sobolev (HLS) type system,

$$\begin{cases} (-\triangle)^k u = v^q, \ u > 0, \ \text{ in } R^n, \\ (-\triangle)^k v = u^p, \ v > 0, \ \text{ in } R^n \end{cases} \tag{1}$$

is a fundamental nonlinear PDE system, and often arises as the "blown-up" equations for many nonlinear problems with the "associated dominating nonlinearity". Together with its special cases, this system is perhaps the most studied PDE system in recent decades. It plays crucial roles in geometric analysis, dynamics analysis of vacuum states, study of nonlinear Schrödinger equations, and many other research areas.

The system can be categorized into three cases depending on parameters p, q and k, namely, subcritical case $\frac{1}{p+1} + \frac{1}{q+1} > \frac{n-2k}{n}$, critical case $\frac{1}{p+1} + \frac{1}{q+1} = \frac{n-2k}{n}$ and supercritical case $\frac{1}{p+1} + \frac{1}{q+1} < \frac{n-2k}{n}$. We propose to study that, the non-existence of positive solution of subcritical cases (i.e. the Lane-Emden Conjecture), the essential uniqueness of solution in the critical cases and the existence and uniqueness of solution in the supercritical cases.

By implementing our new or improved methods in analysis, we hope to obtain a deeper understanding and hence make a small step forward to solving the above fundamental problems, which also may lead to better understandings of various PDE problems arising from different research areas in science and engineering.

References

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