Poster Session A
Thursday Jan. 4, 7:30pm-9:00pm


Bedros Afeyan, Polymath Research, Richard Sydora, U. Alberta; Bradley Shadwick, U. Nebraska-Lincoln, Jeffrey Hittinger, LLNL, David Larson (LLNL)

Self-consistent, collective dynamics of electrons in the presence of non-stationary electric fields is considered. Waves can be driven resonantly (linear, single mode response) and non-resonantly (non-linear, multi-mode response). We examine their individual dynamics using Vlasov (phase space grid and particle in cell (PIC)) simulations and their mutual interactions. We find complex structure formation such as nested partitions in phase space each with its own collective dynamics; trapping-untrapping-retrapping oscillations; order out of chaos; and examples of self-organized states, far from equilibrium, through the action of intense fields. This is a hallmark of high energy density plasma physics. Reduced models for and generalizations of electron plasma waves (EPW) and Kinetic Electrostatic Electron Nonlinear (KEEN) waves will be given. The prospects for sculpting complex phase space structures nonlinearly, using external electromagnetic coherent fields, in order to control plasma and radiation sources of the future, will be discussed.

A2 - A data-driven approach to computing time-dependent active subspaces in dynamical systems.

Izabel P. Aguiar, Paul G. Constantine, University of Colorado, Boulder.

Active subspaces identify important linear combinations of parameters, enabling more powerful and effective dimension reduction. Although active subspaces give model insight and computational tractability for scalar-valued functions, it is not enough. This analysis does not extend to time-dependent systems. Extending active subspaces to time-dependent systems will enable uncertainty quantification, sensitivity analysis, and parameter estimation for computational models that have explicit dependence on time.

The state-of-the-art method for identifying time-dependent active subspaces is to compute them at individual time steps. Using this approach we identify active subspaces in various engineering and biological dynamical systems. This approach is computationally expensive, however: it requires resampling, computing, and decomposing at every time step. In rapid transients, necessarily small time steps lead to many more computations.

To reduce computational cost we implement Dynamic Mode Decomposition (DMD) [Kutz et al., 2016] and Sparse Identification for Nonlinear Dynamical Systems (SINDy) [Brunton et al., 2016] to reconstruct and predict future active subspaces. We also derive analytical forms of time-dependent active subspaces for time-dependent outputs of two linear parameterized dynamical systems. This analysis and computation inform visualization and insight of parameter dependence in various dynamical systems.

A3 - Universality of the Evolution in Paradigmatic Mixed Systems

Or Alus, Shmuel Fishman, Technion.

We discuss a statistical theory for Hamiltonian dynamics with a mixed phase space, where in some parts of phase space the dynamics is chaotic while in other parts it is regular. Transport in phase space is dominated by sticking to complicated structures and its distribution is universal. The survival probability in the vicinity of the initial point is a power law in time with a universal exponent. We calculate this exponent in the framework of the Markov Tree model proposed by Meiss and Ott in 1986. It turns out that, inspite of many approximations, it predicts important results quantitatively. The calculations are extended to the quantum regime where correlation functions and observables are studied.

A4 - Cluster synchronization of diffusively-coupled nonlinear systems: A contraction based approach

Zahra Aminzare, Biswadip Dey, Elizabeth N. Davison, Naomi Ehrich Leonard, Princeton University.

Finding the conditions that foster synchronization in networked oscillatory systems is critical to understanding a wide range of biological and mechanical systems. However, the conditions proved in the literature for synchronization in nonlinear systems with linear coupling, such as has been used to model neuronal networks, are in general not strict enough to accurately determine the system behavior. We leverage contraction theory to derive new sufficient conditions for cluster synchronization in terms of the network structure, for a heterogenous network where the intrinsic nonlinear dynamics of each node may differ. Our result requires that network connections satisfy a cluster-input-equivalence condition, and we explore the influence of this requirement on network dynamics. For application, we implement sufficient conditions for synchronization in networks of FitzHugh-Nagumo oscillators with electrical gap junction coupling and reduce large networks of neurons to corresponding representative networks that are more tractable to analysis. Improving the analytical conditions for when cluster synchronization will occur based on network configuration is a significant step toward facilitating understanding and control of complex oscillatory systems.
A5 - Forecasting Events in the Complex Dynamics of a Semiconductor Laser with Optical Feedback

Andres Aragoneses Aguado, Meritxell Colet, Carleton College.

Nature presents many fascinating complex systems in which characterizing the appearance of extreme events is tantamount. Examples of these include earthquakes, rogue waves, neuronal spikes activity, social networks, variable stars, etc. Forecasting these extremes is extremely challenging, not only because of the presence of noise, but also, because of lack of information about the system. Semiconductor lasers with feedback are a controllable example that present this event-type behavior that resembles other complex system’s dynamics, such as neuronal spikes [1].

We study the complex dynamics of the output power of a semiconductor laser with optical feedback in the LFF (low frequency fluctuations) regime, where sudden dropouts of intensity are present in the output intensity of the laser. We find that the power dropouts show two different dynamics competing. These two types of dropouts can be distinguished based on the depths of the dropouts., i.e., a shallow, more deterministic dynamics, and a deep, more stochastic dynamics, contrary to what would be expected as experimental noise should be stronger in sallower dropouts. As we increase the pump current of the laser we find three different regimes: (i) for low pump currents, at the onset of the LFF region two behaviors compete, (ii) for intermediate pump currents, where the LFFs are well developed, the deeper dynamics dominate, and (iii) for higher pump currents, at the transition from LFFs to coherent collapse a dual dynamics takes place again.

We use the ordinal patterns (OP) analysis to investigate the dynamics. This analysis transforms the time series of events into pattern that unveil temporal and intensity correlations among consecutive events. We use the OPs to predict when the extreme events of the deeper dynamics occur. We study the probabilities of the different OPs that take place right before an extreme event. We find that, in the transition regimes, when two behaviors are competing, we can predict the occurrence of the extreme events more than 95% of the times. For the well developed LFFs regime, where most of the events are due to the same type of behavior, our predictability drops to about 65%.

A6 - Is Human Atrial Fibrillation Stochastic or Deterministic?

Konstantinos N Aronis, Susumu Tao, Hiroshi Ashikaga, Johns Hopkins University.

A reason why the mechanism that maintains human atrial fibrillation (AF) remain unclear may derive from the stochasticity of cardiac dynamics. Using forbidden ordinal patterns (FOP) with Bandt-Pompe symbolization, we characterize the intracardiac bipolar electrograms of AF from 15 patients. We calculate normalized Shannon entropy and Jansen-Shannon statistical complexity. We compare the results with stochastic time series as well as simulated bipolar spiral waves with different levels of white Gaussian noise. For all series, we construct surrogate data with the same frequency spectrum and autocorrelation, and estimate the ratio of FOP decay in the first 6,000 timepoints. Human AF exhibit a median of 460 FOP (range 357-525) which is significantly higher from surrogate data (range 0-103, p < 0.05) and all stochastic series (range 0-5, p < 0.05). The ratio of FOP decay in time series over surrogate data is significantly lower in AF compared to any stochastic series. On the causal entropy-complexity plane, human AF is of lower entropy, higher complexity than stochastic series, and of higher entropy, higher complexity than simulated spiral waves. We conclude that, although human AF is quantitatively different from simulated spiral waves, it has little evidence to suggest stochasticity.

A7 - Localizing rotors in human atrial fibrillation using differential entropy

Konstantinos N Aronis, Susumu Tao, Hiroshi Ashikaga, Johns Hopkins University.

Rotors sustain atrial fibrillation (AF) and are targeted during AF ablation with variable results. Although Shannon entropy was shown to identify rotors of spiral waves in experimental settings, it fails to detect rotors in human atrial fibrillation (AF). We previously showed that the accuracy of identification sensitively depends on the number of rotors and spatial resolution of signal recording. Another possible limitation of the approach is discretization of probability distribution from continuous unipolar electrograms. Therefore, we assessed the value of differential entropy (DE), which does not require discretization of probability distribution, to locate rotors of human AF using intracardiac 64-electrode basket catheter recordings from 33 patients. The area of maximum DE and rotors precisely overlapped in 9% of the recordings, and was one electrode away in 36%. When the maximum DE was defined as the highest 5th percentile, the area of maximum DE and rotors precisely overlapped in 22% of the recordings, and was one electrode away in 59%. We conclude that DE is a promising metric to locate rotors in human AF, but the accuracy is modest.

A8 - X-band Microwave Accelerating Cavity

Joseph Betz, Brian Beaudoin, Widener University.

Inspired by research conducted at Stanford’s Linear Accelerator Center (SLAC), this research focused on the designing and manufacturing of an X-band (8 - 12 GHz) microwave accelerating cavity. Conventional RF accelerators are manufactured by molding sheet metal to the desired dimensions of the cavity and brazing the pieces together. Advances in manufacturing technology such as CNC mills, lathes, and 3D printers have allowed smaller and more precise cavities to be created. The accelerating cavity presented here, was designed using CAD software and optimized using finite element analysis (FEA) simulations. An aluminum test cavity was then manufactured for experimental testing. Various parameters were calculated using simulation software and compared to the experimental results. The completed accelerator would consist of many of these cavities arranged along a beam tube. Future
work on this project will consist of creating a multi-cavity designs which include a series of waveguides and apertures to deliver power to each cavity.

**A9 - Emergent Bistability and switching in a nonequilibrium crystal**

Justin C. Burton, Guram Gogia, Emory University.

Multistability is an inseparable feature of many physical, chemical, and biological systems which are driven far from equilibrium. In these nonequilibrium systems, stochastic dynamics often induces switching between distinct states on emergent time scales; for example, bistable switching is a natural feature of noisy, spatially extended systems that consist of bistable elements. Nevertheless, here we present experimental evidence that bistable elements are not required for the global bistability of a system. We observe temporal switching between a crystalline, condensed state and a gas-like, excited state in a spatially-extended, quasi-two-dimensional layer of charged microparticles. The switching occurs over a broad range of time scales. Nevertheless, a dominant time scale is set by an external, periodic signal, such as a rotation of the crystal due to an external magnetic field. Increasing the number of particles in the system results into more complex, avalanche-like dynamics - only part of the system switches from crystalline to gaseous phase, and the size of the switching region varies with each avalanche event. We confirm our results using molecular dynamics simulations which show that conservative forces, damping, and stochastic noise are sufficient to induce switching.

**A10 - Control of Unknown Chaotic Systems with Reservoir Computing**

Daniel M. Canaday, Aaron Griffith, Daniel Gauthier, Ohio State University.

We present a recurrent neural network (RNN) method for controlling an unknown dynamical system. The approach is based on a type of RNN called a reservoir computer (RC) that is distinguished by an untrained, recurrent hidden layer (called the reservoir) and a memoryless output layer that can be rapidly trained by a simple linear regression. We demonstrate that RCs can learn a relationship between an unknown system’s inputs and outputs and that this relationship can be used for control. We demonstrate the utility of this method by stabilizing the well-studied Lorenz-63 system with noise to fixed points and to periodic orbits.

**A11 - A topological foundation of the present-day “monsoon climate” regime on Earth**

Peter Carl, ASWEX - Applied Water Research.

Projections on the 1991 Kuwait oil well fire problem using a coarse resolution, yet temporally and physically resolved, General Circulation Model of the atmosphere (AGCM) of the Mintz-Arakawa family, led to the understanding of the boreal summer monsoon as dynamic core of a distinct global climate regime borne in topological changes of the seasonal cycle between spring and autumn bifurcations. The ‘architecture’ of these intraseasonal dynamics has been carved out using a large set of conceptual simulations aimed at probing the GCM’s attractor set and its impact on both intraseasonal and interannual climate dynamics. Best visible in variables that relate to global integrals of motion (interhemispheric exchange of atmospheric mass, angular momentum of the atmosphere, enstrophy), the GCM’s dynamic ‘skeleton’ of the season turns out to consist of a complete, yet inverse, route to planetary-scale “monsoon chaos” - to which the system is kicked at monsoon onset by a hard jump when passing a subcritical Hopf bifurcation in boreal spring. The seasonal cycle inflates into a torus segment there, i.e. develops a second, independent oscillation which is not astronomically driven but rather a self-maintained natural mode, organized in a complex, planetary-scale feedback cycle - the well-known 30-60 day monsoon activity cycle. The very existence of this regime of the ‘interactive’ monsoon, with its inherent tropical-extratropical interactions, may be traced back to elementary conditions in the status of two of the global variables used to represent it: atmospheric mass balance between the hemispheres and sub-, co- or superrotational status of the atmosphere with respect to the surface (of a model Earth without dynamic ocean and other contributors to the real-world angular momentum). As the season advances, the backwards running period multiplication scenario reaches a period-one oscillation via changing synchronizations of planetary waves (at lowest-possible rational frequency relationships, indicating a high degree of internal synchronization) and ends up in a wrinkling torus segment before it ceases to exist at the date of monsoon retreat. The wrinkle itself originates in the remainder of an ‘enstrophy stage’ at the transition from period-two to period-one and is the host then of a slow, degenerate oscillation after monsoon retreat: an irregular wander between summer and winter fixpoints which shows up marked signs of the Southern Oscillation. Apart from the attractor studies addressing the topological aspects of present-day monsoon dynamics in boreal summer, short- and longer-term seasonal runs under varied external conditions (i.e. changing insolation at top of the atmosphere) show the impact of the intraseasonal dynamic object uncovered. Three monsoon retreat categories may be distinguished (termed as of “La Nina prone”, “Indian summer” and “El Nino trigger” type) which make up the autumn climate, stamp their signatures on the GCM’s interannual to centennial climate dynamics, and may generate different variability regimes under changing conditions. Paleoclimatic aspects are also related to the existence or non-existence of the ‘interactive’ monsoon regime. The GCM’s high sensitivity to the smoke clouds rising in 1991 in the neighborhood of the strongest equator-crossing circulation, the Somali Jet (which feeds the South Asian monsoon system in boreal summer), thus set the trace to a topologically founded monsoon hypothesis.
A12 - Cutting and Shuffling with Diffusion: Cut-offs in Interval Exchange Maps

Ivan C. Christov, Mengying Wang, Purdue University.

“Cutting and shuffling” underlies mixing in tumbled granular flows. Mathematically, cutting and shuffling can be represented by piecewise isometries on the sphere, or by interval exchange maps on an interval of the real line. We study a class of interval exchange maps (with two parameters: number of subsegments, and a subsegment length ratio), presenting a comprehensive computational study of finite-time mixing by cutting and shuffling. Additionally, we include diffusion into the process by alternating between the interval exchange and a running average across the line, which emulates diffusion. Specifically, we seek to address whether “cut-offs,” previously observed in card shuffling (Aldous and Diaconis, 1986) exist in these simple interval exchange maps with diffusion. To do so, we consider a standard set of $L^p$ norms and measure mixing through the $L^p$ norm of the difference between the distribution of “color” along the segment, compared to the average. The evolution of the norm is averaged over all possible “interesting” permutations (specifically, those that are irreducible, not rotations, and do not exchange adjacent elements) for a given number of subsegments and a subsegment length ratio. Following Liang and West (2008), a critical “half-mixing” time, which corresponds to the number of iterations to the cut-off, is identified for each averaged concentration curve of the cutting and shuffling protocol with diffusion. Then, all of the latter are rescaled onto a “universal” mixing curve. Within some computed error bars, this rescaling holds for several choices of the number of subsegments and the subsegment length ratio, confirming said universality and proving computational evidence for cut-offs in one-dimensional interval exchange maps with diffusion. Thus, much like it always takes 7 riff s to essentially randomize a deck of 52 cards (Bayer and Diaconis, 1992), mixing by cutting and shuffling with diffusion exhibits universality with the “necessary” number of iterations of cutting and shuffling with diffusion scaling strongly only with the number of subsegments in the interval exchange and the subsegment length ratio but not with the specific permutations (shuffles).

A13 - The tipping point: a mathematical model for the profit-driven abandonment of restaurant tipping

Sara M. Clifton, University of Illinois at Urbana-Champaign; Eileen Herbers, Jack Chen, Daniel M. Abrams, Northwestern University.

The custom of voluntarily tipping for services rendered has gone in and out of fashion in America since its introduction in the 19th century. Restaurant owners that ban tipping in their establishments often claim that social justice drives their decisions, but we show that rational profit-maximization may also justify the decisions. Here, we propose a conceptual model of restaurant competition for staff and customers, and we show that there exists a critical conventional tip rate at which restaurant owners should eliminate tipping to maximize profit. Because the conventional tip rate has been increasing steadily for the last several decades, our model suggests that restaurant owners may abandon tipping en masse when that critical tip rate is reached.

A14 - Stability of traveling pulses in a FitzHugh-Nagumo system via the Maslov index

Paul G. Cornwell, Chris Jones, University of North Carolina at Chapel Hill.

The Maslov index is a topological invariant assigned to curves of Lagrangian planes. It has been used in recent years to understand the spectra of linear operators, often arising in the stability analysis of nonlinear waves. In the context of waves, the Maslov index can be thought of as a generalization of Sturm-Liouville theory to systems of equations. One issue plaguing the application of the Maslov index to stability problems is that it is difficult to calculate in practice. This typically entails tracking the tangent space to an invariant manifold as it moves around phase space. We address this issue by analyzing traveling pulses in a FitzHugh-Nagumo system. The timescale separation in the traveling wave equation allows us to reduce the dimension of the problem and make the calculation of the index tractable. We use this calculation in conjunction with a recent result relating the Maslov index to stability in skew-gradient systems to prove that the fast traveling pulses are stable.

A15 - Analytic Solutions throughout a Period-Doubling Route to Chaos


We describe a class of dynamical systems that can be analytically solved throughout a period-doubling route to chaos. These oscillators consist of a linear equation, which describes growing oscillations about a set point, and a nonlinear switching rule that periodically updates the set point. Previously it has been shown that similar systems can be analytically solved even when the oscillations are chaotic. However, the prior solvable systems were limited to robust chaos and exhibited few bifurcations, transitioning directly from a steady state to chaos. Here we show that a simple change to the update rule allows for a greater variety of nonlinear phenomena, including a period-doubling route to chaos, while maintaining exact analytic solvability. Two specific examples are shown. The first is a pedagogical oscillator, for which the set point update rule is defined such that a return map is conjugate to the logistic map. With this oscillator, an exact solution is provided throughout an entire period-doubling route to chaos. The second example is an electronic circuit, which experimentally confirms analytic solvability throughout a period-doubling route to chaos in a physical system. These results suggest that analytic solutions may be more useful in studying nonlinear dynamics than previously recognized. This work is published in Physical Review E 95, 062223 (2017).
A16 - A Geometric phase transition in physical networks embedded in 3D

Nima Dehmamy, Soodabeh Milanlouei, Laszlo Barabasi, CCNR Northeastern University.

In many physical networks, from neurons in the brain to 3D integrated circuits or underground hyphal networks, the nodes and links are physical objects unable to cross each other. These non-crossing conditions constrain their layout geometry and affect how these networks form, evolve and function, limitations ignored by the extensive theoretical framework used to characterize real networks. Indeed, the currently used network layout methods are variants of the Force-Directed Layout (FDL) algorithm, which assumes dimensionless nodes and links, hence are inherently unable to reveal the geometry of densely packed physical networks. Here, we develop a modeling framework inspired by manifold dynamics that accounts for the physical reality of nodes and links, allowing us to explore how the non-crossing conditions affect the geometry of the network layout. For small link thicknesses, we observe a weakly interacting phase where the layout avoids the numerous potential link crossings via local link rearrangements, without altering the overall layout geometry. Once exceeds a critical threshold, a strongly interacting phase emerges, where multiple geometric quantities, from the total link length to the link curvature, scale with . We show that the observed transition is driven by excluded volume interactions, allowing us to analytically derive the transition point. We also find that networks display a solid-like response to stress in the weakly interacting phase, whereas they behave in a gel-like fashion in the strongly interacting phase, illustrating the impact of the observed transition on the physical properties of the network. Overall, we observe a deep universality, finding that the observed scaling properties are independent of the underlying network topology. Finally, we show that the weakly interacting phase offers avenues to 3D print networks, while the strongly interacting phase offers insight on the scaling of densely packed mammalian brains.

A17 - Feedback Controlled Bifurcation of Evolutionary Dynamics with Generalized Fitness

Biswaip Dey, Naomi Ehrich Leonard, Princeton University; Alessio Franci, National Autonomous University of Mexico; Kayhan Ozcimder, The MathWorks, Inc., Natick, MA, USA.

Coexistence and interaction of multiple strategies in a large population of individuals can be observed in a variety of natural and engineered settings. In this context, the replicator-mutator dynamics model the evolution of fractions of a population committed to different strategies as a function of two counteracting factors - replication (commitment to strategies with higher rewards/pay-offs) and mutation (tendency for spontaneous switch among strategies). As these dynamics provide an efficient tool to analyze the evolution of the fractions of the total population committed to different strategies, they have been used to study a variety of problems (e.g. opinion formation in social-networks, decision-making in multi-agent systems, evolution of language grammars, population genetics in biology). Although the literature addresses existence and stability of equilibrium points and limit cycles of these dynamics, linearity in the fitness functions has typically been the underlying assumption. Here we propose a further generalization of these dynamics by introducing a nonlinear fitness function that enriches the steady-state behavior of the dynamics. We show that under certain conditions the replicator-mutator dynamics for two competing strategies exhibit a symmetric quintic bifurcation, i.e. a pitchfork bifurcation with a quintic (instead of a cubic) stabilizing term, and a symmetric subcritical pitchfork bifurcation in its unfolding. The latter provides a hysteresis that we leverage by designing slow-time-scale feedback dynamics for the mutation rate, i.e. the underlying bifurcation parameter. By using techniques from geometric singular perturbations, we show that the closed-loop dynamics produce oscillatory behaviors in the evolution of fractions of population committed to two strategies. We also notice that any asymmetry in the associated payoff structure causes the subcritical pitchfork to unfold into one of its persistent forms. We leverage this aspect by designing ultraslow-time-scale dynamics to control asymmetry in the payoff structure, i.e. the associated unfolding parameter, and demonstrate an even richer class of behaviors. These results show a range of ways in which nonlinear dynamics and bifurcation theory can be used as a constructive means to design closed loop adaptive dynamics at multiple time scales.

A18 - Stability of Spiral Waves in Cardiac Dynamics

Stephanie Dodson, Bjorn Sandstede, Brown University.

Ventricular tachycardia, a dangerous fast-paced heart rate, is a result of a sustained spiral wave rotating on the surface of the heart. After the spiral destabilizes, unorganized electrical activity leads to sudden cardiac arrest - the leading natural cause of death in the US. I will present stability results of spiral waves formed in the Karma Model, which is a reaction-diffusion system describing electrical activity in cardiac tissue. My talk will highlight spectral properties of spiral waves formed in bounded disks in reaction-diffusion systems. Absolute and essential spectra of spirals are calculated using asymptotic wave trains, and are compared with point spectra of spirals on large disks. In addition, I will address difficulties that arise in spectral calculations when one or more components of the system are diffusionless.

A19 - Control of macroscopic neuronal network activity is optimal at criticality

Kathleen Finlinson, Daniel B. Larremore, Juan G. Restrepo, University of Colorado at Boulder.

There is growing evidence that the cortex operates at a critical state where the strength of excitatory and inhibitory neurons is precisely balanced. Recent research has shown that many properties related to information processing are optimized at criticality, including dynamic range, information transmission, and the variability
of synchronization. By analyzing neuronal network models, we show here that, in addition, the macroscopic activity of the network is most easily controlled at criticality.

We first analyze a simple binary neuron model, where nodes may be either active or quiescent at each time. The balance in the relative numbers and synapse strengths of excitatory and inhibitory nodes determines whether excitations are amplified, maintained, or decay on average (corresponding to supercritical, critical, and subcritical states, respectively). We consider the problem of controlling the total network activity to a given target value, using a feedback loop with either global or local information. We show numerically and theoretically that the control error, averaged across a range of targets, is minimized when the network is critical. We validate our findings for the binary neuron model by simulating a conductance-based neuron model.

A20 - Predicting Full and Partial Synchronization of Kuramoto Oscillators on Arbitrary Networks

Brady M. Gilg, Dieter Armbruster, Arizona State University.

The network Kuramoto model is a simple model of phase oscillators embedded in a network. Each oscillator is assumed to have nonidentical internal dynamics, as well as external forcing mediated by a coupling function from each oscillator to its neighbors. Two fascinating emergent phenomena exhibited by the model are those of “full synchronization” and “partial synchronization”, occurring at higher and lower strengths of coupling. In the former, all oscillators maintain an identical constant frequency. In the latter, the Kuramoto network segments into clusters of oscillators which are frequency synchronized within themselves, but not to external clusters. The current literature covers a number of results which establish upper bounds on the critical coupling required for full synchronization, but very few lower bounds, which are frequency synchronized within themselves, but not to external clusters. The current literature covers a number of results which establish upper bounds on the critical coupling required for full synchronization, but very few lower bounds, and nearly no results pertaining to partial synchronization. Here, we introduce a Kuramoto network reduction technique and show how this reduction leads to a weighted graph partitioning problem. The partitioning problem can then be solved approximately with known optimization techniques utilizing spectral graph theory. Finally, we provide numerical evidence that this technique produces a lower bound on the critical coupling and predicts the partially synchronized behavior of a Kuramoto system with high accuracy.

A21 - Capturing the complexity of clouds

Franziska Glassmeier, National Research Council and NOAA; Fabian Hoffmann, Takanobu Yamaguchi, Jan Kazil, NOAA and CIERES, University of Colorado at Boulder; Jill Johnson, Ken Carslaw, University of Leeds, Leeds (UK); Graham Feingold, NOAA.

Clouds present the single biggest unknown in our ability to predict future climate states. Cloud systems are highly complex and governed by multiple feedbacks. They are usually studied with numerical models at a range of scales and levels of detail. These models are constrained by observations from in-situ measurements and remote sensing.

Global climate models, our main tool for climate projections, are run on meshes that are too coarse to resolve important cloud processes, while high resolution models are too computationally expensive to be applied at climate-relevant spatiotemporal scales. Poor representation of clouds in global models translates to an uncertainty in whether cloud system responses will enhance or offset temperature changes associated with increases in greenhouse gases.

We will present a collection of studies that approach the cloud problem from the perspective of low-dimensional chaotic systems, network analysis and Gaussian process emulation. These studies elucidate the fundamental behavior of cloud systems, including attractive states, bifurcations and pattern formation. By capturing the complexity of clouds in these ways, we aim to improve their representation climate models, thereby reducing the uncertainty in cloud feedbacks and climate projections.

A22 - Linking group theory and dynamical systems analysis via invariant manifolds

Keshlan Govinder, University of KwaZulu-Natal, South Africa

There are a various approaches to finding analytic solutions to, and determining properties of solutions of, differential equations. One can apply (amongst others) the very successful method of group analysis (which exploits the invariance of equations under transformations) to classify and find explicit solutions of differential equations. Singularity analysis (which requires solutions of equations to have, at worst, moveable poles) has also been used to classify equations. The long term evolution of systems of equations can be determined by application of dynamical systems analysis (where the behaviour of equilibrium points is paramount).

For some time now, imposing the requirement of invariance under suitable point transformations has restricted parameter values in certain differential equations. Interestingly, these restricted values coincide with those obtained from singularity analysis. However, there seems to be a disconnect between those theories and dynamical systems analysis. In particular, bifurcation values do not coincide with the restricted values from group and singularity analysis.

In this talk we will explore the link, if any, between group analysis and linearisation around fixed points for certain problems. We will also comment on the role of transformations in these synchronous/complementary results and show that some known results may need to be reinterpreted.

A23 - Galaxy-Like organization of floaters on Faraday Waves

Pablo Gutiérrez, Universidad de O’Higgins, Chile, H. Alarcón, N. Perinet, and N. Mujica, Universidad de Chile; L. Gordillo, Universidad de Santiago, Chile.

In experiments of standing waves in a channel, we observe a complex pattern of tiny floating particles. Starting with an homogeneous distribution of floating
particles, we drive the container with a vertical oscillation. After standing waves develop (because of Faraday instability), particles start to agglomerate in elliptical patterns with arms on the sides. The arms structure resembles the one encountered in some Galaxies.

Floater patterns appear in well defined positions in the channel, around the anti-nodes of the waves. Patterns rotate with a well defined velocity that scales with wave amplitude. The arms also grow with the wave amplitude, despite they appear at a threshold different from the one of the circulation. We interpret our observations as follows: viscosity drives a steadily circulating streaming flow [with the same origin as in Perinet et al. JFM, 2017]. The circulation couples with the periodic flow induced by the standing wave, generating the folding of the material lines in the fully. One arm appears after each wave period.

Although standing waves are not supposed to transport matter, it was reported that floating particles accumulate on the nodes or anti-nodes of standing waves, depending on the wetting properties and inertia of particles [Falkovich et al. Nature, 2005]. Latter, in similar experiments it was observed that the covering fraction of the surface is also relevant to trigger more complex collective scenarios [Sanli et al. PRE 2014]. Aside from these mechanisms related to the physical properties of particles, our observations suggest a mechanism more related with the geometry of the flow itself.

**A24 - Universal Scaling Properties of Cooperative Contagion with Structural Heterogeneity**

Meesoon Ha, Chosun University, Korea; Kihong Chung, KAIST, Korea; Yongjoo Baek, University of Cambridge, UK; Daniel Kim, Samsung SDS, Korea; Hawoong Jeong, KAIST, Korea;

We investigate universal scaling properties of the generalized epidemic process (GEP) on random scale-free networks. The GEP is a simple model of contagion with cooperative effects, which give rise to tricritical percolation transitions. Using the standard tree approximation, we identify the relevant length scales governing the tricriticality and derive their scaling exponents as functions of the degree exponent, identifying crossover behaviors caused by the competition between the length scales. Interestingly, we find that the maximum degree sets an important length scale on its own, which induces nontrivial cutoff-dependent finite-size effects for a broad range of degree exponents. We propose an extended finite-size scaling theory encompassing these properties, which are confirmed with extensive Monte Carlo simulations.

**A25 - Tracking hidden nodes in networks using adaptive filtering**

Franz Hamilton, North Carolina State University.

In network analysis, we are often confronted with the problem of hidden nodes, or parts of a network that we have no measurements, and thus no knowledge, of. These hidden nodes act as unknown drivers and can have a significant influence on the network state. Detecting their presence, or more specifically the parts of the network they are influencing, is thus critical. We propose an adaptive filtering-based method for the detection and tracking of hidden nodes in complex networks. By viewing the hidden nodes as a problem of missing variables in the model fitting process, we show that the filter estimated system noise covariance allows us to localize the effects of the hidden node and track changes in its driving strength over time.

**A26 - Localization of Laplacian eigenvectors on random networks**

Shigefumi Hata, Kagoshima University; Hiroya Nakao, Tokyo Institute of Technology.

In large random networks, each eigenvector of the Laplacian matrix tends to localize on a subset of network nodes having similar numbers of edges, namely, the components of each Laplacian eigenvector take relatively large values only on a particular subset of nodes whose degrees are close [1-3]. Although this localization property has significant consequences for dynamical processes on random networks, a clear theoretical explanation has not yet been established. In this study we analyze the origin of localization of Laplacian eigenvectors on random networks by using a perturbation theory. We clarify how heterogeneity in the node degrees leads to the eigenvector localization and that there exists a clear degree-eigenvalue correspondence, that is, the characteristic degrees of the localized nodes essentially determine the eigenvalues. We show that this theory can account for the localization properties of Laplacian eigenvectors on several classes of random networks, and argue that this localization should occur generally in networks with degree heterogeneity [4].

**References**


**A27 - A Simple ODE Model for Advertising Dynamics**

Joseph D. Johnson, Daniel M Abrams, Northwestern University.

Firms in the U.S. spend over 150 billion dollars a year in advertising their products to consumers. Given the magnitude of spending, it is of great interest to understand how that budget could be optimally distributed, how market prices should be affected. Using a simple dynamical systems model, we explore firm behavior under idealized conditions and find a surprising prediction: firms should split into two groups, one with significantly less advertising (a "generic" group) and one with significantly more advertising (a "name-brand" group). We use consumer data to compare predictions from the model with real world pricing and find qualitative agreement.
A28 - In silico analysis of antibiotic-induced C. difficile infection

Eric Jones, Jean Carlson, UC Santa Barbara.

We study antibiotic-induced C. difficile infection (CDI), caused by the toxin-producing C. difficile (CD), and implement clinically-inspired simulated treatments in a computational framework that synthesizes a generalized Lotka-Volterra (gLV) model with SIR modeling techniques. The gLV model uses parameters derived from an experimental mouse model, in which the mice are administered antibiotics and subsequently dosed with CD. We numerically identify which of the experimentally measured initial conditions are vulnerable to CD colonization, then formalize the notion of CD susceptibility analytically. We simulate fecal transplantation, a clinically successful treatment for CDI, and discover that both the transplant timing and transplant donor are relevant to the efficacy of the treatment, a result which has clinical implications. We incorporate two nongeneric yet dangerous attributes of CD into the gLV model, sporulation and antibiotic-resistant mutation, and for each identify relevant SIR techniques that describe the desired attribute. Finally, we rely on the results of our framework to analyze an experimental study of fecal transplants in mice, and are able to explain observed experimental results, validate our simulated results, and suggest model-motivated experiments.

A29 - Information Anatomy of Printed English

Alexandra M. Jurgens, James P. Crutchfield, University of California at Davis.

The Shannon entropy rate is commonly used in quantitative linguistics to characterize and compare natural languages. As developed by Shannon, though, entropy describes the compressibility of a stochastic source. In particular, it does not alone identify the mechanisms by which natural language can be generated (written) and predicted (perceived). Appealing to multivariate information theory, we dissect the Shannon entropy of printed English into its information atoms-bound, ephemeral, excess, elusive, and related informations-offering linguistic interpretations of these measures. Using empirical estimates derived from massive text corpora, we show that the measures shed light on the internal information dynamic of natural language, allowing us to investigate structure and stationarity. We also identify information shared between past and future text not contained in a present letter, which implies the existence of hidden internal states in natural language.

A30 - Low-dimensional dynamics in an attractor model of working memory interference

Zachary Kilpatrick, University of Colorado at Boulder.

Information from preceding trials of cognitive tasks can bias performance in the current trial, a phenomenon referred to as interference. Subjects performing visual working memory (WM) tasks exhibit interference in their responses: the recalled target location is biased in the direction of the target presented on the previous trial. We present modeling work that links WM interference to computations of a recurrent network wherein short-term facilitation (STF) accounts for the observed bias. WM of an angle around a circle (0 ≤ θ ≤ 360) is stored by a localized region of neural activity called a bump. Network connectivity is reshaped dynamically during each trial, generating predictions from prior trial observations. Applying timescale separation methods, we approximate the low-dimensional dynamics of the bump as a particle in a slowly changing potential on the periodic domain θ ∈ [0, 360). Furthermore, we demonstrate that STF results in stochastic dynamics of the bump that exhibit two timescales, a fast timescale corresponding to initial free diffusion of the bump followed by a slow timescale controlled by the rate of the facilitation process.

Our neuronal network model is comprised of two variables evolving in space x ∈ [0, 360]°, corresponding to the angle preference of neurons at that location, and time t > 0. The evolution equations consist of one stochastic integrodifferential equation and one auxiliary differential equation:

\[ \dot{u}(x, t) = -u(x, t) + w(x) \ast [(1 + q(x, t))F(u(x, t))] \right) + I(x, t) + Z(x, t), \]
\[ \tau \dot{q}(x, t) = -q(x, t) + \beta F(u(x, t))(q_+ - q(x, t)), \]

where u(x, t) describes neural activity variable. Baseline recurrent coupling is defined by the weight function w(x − y), and the nonlinearity F(u) ∈ [0, 1] converts population input to an output firing rate. Recurrent coupling is shaped by STF in active regions of the network (F(u) > 0), as described by q(x, t) ∈ [0, q_+]; q_+ > 0 and β determine the maximal increase in synaptic utilization and the rate at which facilitation occurs. Effects of the angle input and the network response are described by the deterministic spatiotemporal input I(x, t) and the white noise process Z(x, t).

This population rate model is analyzed to link the architecture of the network to a low-dimensional description of the dynamics of a bump attractor. The position of the bump θ(t) of neural activity u(x, t) corresponds to the location of a particle on a periodic domain [0, 360], while the STF variable q(x, t) generates a spatially heterogeneous potential that influences the stochastic dynamics of θ(t). The remaining facilitation from previous trials produces a heterogeneity in this potential that attracts θ(t), ultimately biasing its long term location in the direction of the previous target angle.

A31 - Interacting bumps model of item limitations in working memory

Nikhil Krishnan, Zachary Kilpatrick, University of Colorado Boulder; Daniel Poll, Northwestern University.

Working memory allows animals to store information for limited periods of time. While working memory can store multiple items simultaneously, increasing the number of items decreases the accuracy of storage. We analyze the dynamics of a neural field model of working memory, which uses stimulus-tuned neurons connected by
The variable $u(x, t)$ represents the synaptic input to spatial location $x \in [-L, L]$ at time $t$, which is periodic $u(L, t) = u(-L, t)$, representing the periodicity of the possible angles used as inputs. The weight function $w(x-y) = A(1-|x-y|)e^{-|x-y|}$ represents the synaptic connectivity (with strength $A$) between neurons at location $y$ and $x$. Multiplicative noise is described by the final term.

Specifically, we study the dynamics corresponding to the retention of multiple visual angles presented simultaneously. Remembered locations are represented by localized regions of persistent activity, called bumps, which are steady states of the noise-free neural field equation. We analyze their dynamics during the memory retention period by developing low-dimensional equations for the bump interfaces, the locations where their activity level equals the firing threshold of the neural field. For instance, the interface equations for the two edges of a single bump can be approximated by the following pair of stochastic differential equations

\[
\tau_2 dx_1 = (\theta - W(x_2 - x_1))dt - \sqrt{\theta}dZ(x_1, t), \\
\tau_2 dx_2 = (-\theta + W(x_2 - x_1))dt + \sqrt{\theta}dZ(x_2, t).
\]

Interface equations for multiple bumps would involve coupling between the interfaces of both bumps. With these effective equations, we can quantify how the relative placement of the visual stimuli impacts the dynamics of the remembered location. In particular, remembered angles can attract or repulse one another depending on their distance from one another. The impacts of stochasticity and network connectivity are also considered.

We demonstrate that greater synaptic strength $A$ increases the upper limit for the capacity of the network. Random fluctuations cause the bump to wander, but an increasing $A$ results in wider bumps which wander less. However, wider bumps have stronger interactions with one another, with greater synaptic strength corresponding to more error caused by attraction and repulsion between bumps. There is a preferred value of synaptic strength which balances bump interaction with the impact of stochasticity, resulting in minimal accumulation of error. As the number of items recalled increases, this optimal synaptic strength progressively decreases.

Michael W. Lee, Earl H. Dowell, Duke University

The novel concept of frequency shredding will be introduced and its applications discussed in the context of a canonical viscous flow. A regularized 2D lid-driven cavity flow was simulated at several Reynolds numbers above the system’s first Hopf bifurcation. The flow was simulated to high enough Reynolds numbers that the flow’s progression through quasi-periodicity to chaos was observed, and with a high enough resolution in Reynolds number that highly Reynolds-sensitive flow states were captured. The first Hopf bifurcation was observed at $Re = 10,250$; by $Re = 15,500$ the flow became dominated by multiple incommensurate frequencies and was therefore characterized as quasi-periodic. The flow then became aperiodic by $Re = 18,000$, but then reverted back to periodicity near $Re = 20,750$. By $Re = 21,250$, the flow reverted back to quasi-periodicity; it then progressed gradually back to aperiodicity as the Reynolds number increased further. These flow transitions were characterized in the power spectrum and verified with Poincaré sections; analyses utilized the previously understood concept of frequency entrainment as well as a novel concept introduced here called frequency shredding. Frequency shredding is observed when a clean peak in the power spectrum breaks down into a collection of nearby frequencies, thus dramatically increasing the likelihood of harmonic interaction elsewhere in the power spectrum. Utilizing only these two concepts, all observed trends in the power spectrum are characterized and explained. As such, the concept of frequency shredding has the potential for characterization and possible prediction of the nonlinear dynamics of high-Reynolds number flows.

A33 - Exact action relations between homoclinic and unstable periodic orbits in chaotic systems
Jizhou Li, Steven Tomovic, Washington State University.

Homoclinic and unstable periodic orbits in chaotic systems play central roles in various semiclassical sum rules. The interferences between terms are governed by the action functions and Maslov indices. In this talk, we identify geometric relations between homoclinic and periodic orbits, and derive exact formulae expressing the periodic orbit classical actions in terms of corresponding homoclinic orbit actions plus certain phase space areas. The exact relations provide a basis for approximations of the periodic orbit actions as action differences between homoclinic orbits with well-estimated errors. This make possible the explicit study of relations between periodic orbits, which results in an analytic expression for the action differences between long periodic orbits and their shadowing decomposed orbits in the cycle expansion.

A34 - Non-normal Dynamics and Nonlinearity in the Feedback Between Motion and Sensation Boosts Run-and-tumble Navigation in Biology
Junjiajia Long, Steven W. Zucker, Thierry Emonet, Yale University.

Many organisms navigate gradients by alternating straight motions (runs) with random reorientations (tumbles), transiently suppressing tumbles whenever attractive signal increases. This induces a functional coupling between movement and sensation, since tumbling probability is controlled by the internal state of the organism which, in turn, depends on previous signal levels. Although a negative feedback tends to maintain this internal state close to adapted levels, positive feedback can arise when motion up the gradient reduces tumbling probability, further boosting drift up the gradient. Importantly, such positive feedback can drive large fluctu-
Autonomous perturbations typically destroy the center of a planar system of differential equations. We show that the transition changes when a cycle of the center is tangent to an impact threshold (i.e., a threshold that generates an impulse). We give an inequality type condition for such a grazing cycle to persist under autonomous perturbations. The result finds an application in neuroscience where it establishes the occurrence of spiking oscillations. The work is supported by NSF CMMI-1436856.

A35 - Grazing bifurcations from a center under autonomous perturbations

Oleg Makarenkov, University of Texas, Dallas

A36 - Controllable Billiards: Characterizing the Paths of Simple Mobile Robots

Alexandra Q. Nilles, Israel Becerra, Steven M. LaValle, Department of Computer Science, UIUC.

Robotic tasks such as localization, navigation, coverage, and monitoring can be expressed as properties of the robot’s path through space. For example, to monitor environmental conditions in a greenhouse, we may want a robot which patrols the space along a repeatable path. We would like to automatically design such paths, while requiring minimal sensing and control on the robot. We note that commercially available robots are able to travel forward in straight lines, stop at boundaries, and turn in place upon reaching such a boundary. Using simple sensors such as proximity detectors, such robots can measure and control their heading angle with respect to environment boundaries. Thus, we are inspired to use billiards and related models to analyse such robotic systems. In particular, we consider robots which always leave a boundary at an angle which is fixed with respect to the normal. This dynamical system is similar to the recently investigated “microorganism billiards” system, but with no sliding along the boundary and the assumption of controllability of the departure angle. Our main result is the existence and exact solutions of stable limit cycles in all convex polygons. For some ranges of departure angles, chaotic dynamics result, and we will show some analysis of these dynamics. We will also show some preliminary ideas for path characterization and synthesis in nonconvex polygons.

A37 - Constrained Stoichiometric Network Analysis

Vuk Radojkovic, Igor Schreiber, CSc., University of Chemistry and Technology Prague, Czech Republic.

Stoichiometric network analysis (SNA) is a method of studying stability of steady states of stoichiometric systems following mass action kinetics by analyzing contributions of elementary subnetworks to stability of the complete network, obtained by linearly combining all elementary subnetworks. In the SNA, reaction rates are expressed using convex parameters (coefficients expressing contributions of elementary subnetworks and reciprocal steady state concentrations), as opposed to kinetic parameters (internal - rate coefficients and external - flow rate, inlet/initial concentrations) used in the standard formulation of the evolution equations. Often the rate coefficients are difficult to obtain by standard experimental methods and a question arises: whether convex parameters can be used for determining unknown kinetic parameters based on available experimental data. Such a task typically does not provide a unique solution because of limited experimental data and a large number of elementary subnetworks. Unfortunately, SNA does not provide any clue for a suitable choice of convex parameters that would lead to kinetic parameters consistent with experimental conditions. We present a systematic method of determining convex parameters consistent with the experimentally known values of system’s variables (measured steady state values) and parameters (external parameters and known rate coefficients) at a bifurcation point, most conveniently at a Hopf bifurcation, giving rise to periodic oscillations. Using the experimental data on the emerging oscillations allows for estimation of unknown rate coefficients by introducing a set of linear constraint equations based on reaction rates expressed as a linear combination of the elementary subnetworks satisfying the experimentally known data. A dominant subnetwork representing a core mechanism leading to Hopf bifurcation is chosen as a reference and its coupling to the rest of the network is step-wise increased. At each step linear programming is used for solving the constraint equations for contributions of the nondominant subnetworks and unknown values of steady state concentrations and rate coefficients. This estimate of unknown kinetic parameters can be further refined by considering more subtle experimentally accessible features, such as data on relative oscillatory amplitudes and/or quenching of oscillations. As an example, we apply the outlined method to the Oregonator model of the Belousov-Zhabotinsky reaction system and demonstrate how both qualitative and quantitative experimental observations can be used for estimating unknown kinetic parameters and identifying a core subnetwork responsible for oscillations while reflecting the experimentally determined and theoretically suggested features of the system.
A38 - On Human Single Leg Quiet Stance

Matthew R. Semak, Patrick T. McMillan, University of Northern Colorado; Gary Heise, School of Sport and Exercise Science, University of Northern Colorado.

The means by which human posture control is maintained is both subtle and complex. Certainly, a great deal of past work by many researchers has afforded us various valuable insights. We have a particular interest in human unipedal balance control. Data collected via a force plate for individuals attempting to maintain upright posture using their dominant and non-dominant legs. The force and jerk of the center-of-pressure for each foot has been examined using power spectral and detrended fluctuation analyses. Both the force and jerk display recurrent behavior on long time scales. One short time scales, the jerk shows persistent correlations which can be modeled by fractional Gaussian noise, and the force is diffusive in nature. Moreover, we attempt to distinguish behavior associated with the dominant leg's dynamics from that of the non-dominant using sample entropy measurements.

A39 - Control of Nonlinear Wave Solutions to Neural Field Equations

Alexander Ziepke, Steffen Martens, Harald Engel, Technische Universität Berlin, Germany.

The investigation of neural fields, describing dynamics of large networks of synaptically coupled neurons by means of continuous field equations, has gained interest over the last decades. In particular, neural field systems exhibit self-organized spatio-temporal structures, such as stationary and traveling fronts and pulses, spiral waves, and localized spot-like bump solutions. This makes them a convenient tool to describe various neural processes, such as working memory, motion perception and visual hallucinations, to name a few. Due to the important applications of neural field models, the question arises how to effectively control solutions in these systems.

In order to address this problem, we extend control techniques, previously derived for reaction-diffusion systems [1], to neural field equations. The proposed open-loop control scheme enables shifting and rotating traveling bump and wave solutions according to a prescribed protocol of motion while simultaneously conserving their shape. Noteworthy, the control signal solely depends on the profile and velocity of the unperturbed solution, and thus, for applying the control, a detailed knowledge of the internal dynamics is not required.

Moreover, we investigate the stability of the suggested scheme and compare it with other recently proposed means of position control such as step-like external inputs and asymmetric spatial coupling kernels.

A40 - Modes of Information Flow: Intrinsic Transfer Entropy

Ryan G. James, James P. Crutchfield, University of California, Davis; Bahtzi Zakirov, College of Staten Island.

Information flow between components of a system can take many forms. We begin by demonstrating three modalities of information flow from time series X to time series Y. Intrinsic information flow exists when the past of X is predictive of the present of Y, independent of the past of Y; this is what is most commonly thought of as information flow. Shared information flow exists when the past of X is predictive of the present of Y in the same manner as the past of Y; this can occur due to synchronization or common driving. Finally, conditional information flow occurs when neither the past of X nor the past of Y are predictive of the present of Y on their own, but together the pasts of X and Y are predictive of the present of Y. The two most common information-theoretic methods of quantifying information flow, the time delayed mutual information and the transfer entropy, are both sensitive to a pair of these modalities: the time delayed mutual information to both intrinsic and shared flow, and the transfer entropy to both intrinsic and conditional flow. In order to quantify each mode individually we introduce a third measure of information flow: the intrinsic mutual information. We then utilize the time delayed mutual information, the transfer entropy, and the intrinsic transfer entropy to quantify the three modalities of information flow in a variety of systems including a hyperchaotic attractor, financial data, and biological data.

Poster Session B
Friday Jan. 5, 7:30pm-9:00pm

B1 - Comparing the Dynamics of Voyager 2 and New Horizons Solar Wind Data
Varad Deshmukh, Liz Bradley, University of Colorado at Boulder.

One of the important problems in planetary science is to understand the dynamics of the solar wind: the stream of charged particles emitted by the sun. A question that is of particular interest is whether those dynamics are stationary. Sensor data from the Voyager 2 and New Horizons missions - launched in 1977 and 2006, respectively - allow us to address this question. In this work, we consider time-series data of solar wind properties (proton density, temperature and velocity) collected at the same heliospheric distance by both missions, roughly 20 years apart. We present a comparative analysis of these properties using nonlinear time-series analysis and information-theoretic techniques. Our findings suggest that the Lyapunov exponents of the density and temperature dynamics are higher in the New Horizons data and lower in the velocity dynamics at the time of the Voyager mission.

B2 - A New Frame for Phase Analysis
Benjamin G. Letson, Jonathan E. Rubin, University of Pittsburgh.

We present a novel coordinate scheme, which we call the Local Orthogonal Rectification (LOR) Frame, with examples which demonstrate its effectiveness. The geometric idea for the LOR frame is simple: given an arbitrary manifold \( M \), which need not be invariant, in a vector field, we project the dynamics into the tangent and normal spaces of \( M \). This allows us to study the dynamics near \( M \) with new precision and to quantify the degree to which \( M \) is invariant. We have used the LOR frame in \( \mathbb{R}^2 \) to provide a precise definition for a “river” in a planar flow, and to study rivers in the wild. We also demonstrate new uses for the LOR frame in three-dimensional flows, which allow us to identify periodic trajectories, canards, and spike adding bifurcations.

B3 - General Proofs and Extensions of the ‘Linear Chain Trick’ for Reducing Integro-differential Delay Equations to ODEs
Paul Hurtado, University of Nevada, Reno; Adam Kirosingh, Stanford University.

Mathematical modelers have long known of a method, referred to as the ‘Linear Chain Trick’ (LCT; aka ‘Gamma Chain Trick’), for reducing some systems of integro-differential equations to a system of ODEs in the special case where these models correspond to mean-field equations for stochastic state transition models with dwell time distributions for each state that obey Exponential or Erlang (Gamma) distributions. Despite the widespread use of this modeling technique, in all but the simplest case we lack general theory to prescribe how to construct such systems of ODEs without first deriving the intermediate mean-field integro-differential equations. Here we provide results that detail how to write down mean-field ODEs directly from underlying stochastic model assumptions without deriving intermediate integro-differential equations. This is achieved via general theorems that extend the LCT to a broader class of scenarios commonly encountered in applications, including extensions for non-Erlang delay distributions. We conclude by presenting a Generalized Linear Chain Trick (GLCT) framework for constructing systems of ODEs for a much broader class of dwell time distributions. These LCT extensions and their proofs (1) clarify the connections between underlying stochastic assumptions and mean-field ODEs, and (2) allow modelers to work beyond the constraints of the LCT to construct mean-field state transition models with ODEs that more accurately incorporate non-Erlang dwell-time distributions.

B4 - Molecular crowding and chromosome-like organization of a ring polymer in a cylindrical space
Youngkyun Jung, Korea Institute of Science and Technology Information; Bae-Yeun Ha, Chanil Jeon, University of Waterloo.

To what extent does a ring polymer resemble a circular bacterial chromosome, often viewed as consisting of “two arms” organized in parallel? Using computer simulations, we show how confinement and molecular crowding organize a ring polymer in a cylindrical space. Our results suggest that in a wide parameter range molecular crowding is essential for separating the two arms, similar to what was observed with circular E. coli chromosomes at fast-growth rates. Under different conditions, however, the ring polymer is centrally condensed or adsorbed onto the cylindrical wall with the two arms laterally collapsed onto each other. We discuss the relevance of our results to chromosome-membrane interactions.

B5 - Atomic Norm Minimization for Modal Analysis from Compressive Measurements
Shuang Li, Dehui Yang, Michael B. Wakin, Gongguo Tang, Colorado School of Mines.

One analytical technique for assessing the health of a structure such as a building or bridge is to estimate its mode shapes and frequencies via vibrational data collected from the structure. A change in a structure’s modal parameters could be indicative of damage. Due to the considerable time and expense required to perform manual inspections of physical structures, and the difficulty of repeating these inspections frequently, there is a growing interest in developing automated techniques for structural health monitoring (SHM) based on data collected in a wireless sensor network (WSN).
In order to save energy and extend battery life, it is desirable to reduce the dimension of data that must be collected and transmitted in the WSN. In recent work by Park et al., which provided a rigorous analysis of a singular value decomposition (SVD) based technique for estimating the structure’s mode shapes in free vibration without damping. As a means of compression, this work considered both random sampling in time and multiplication by random matrices. While promising, the SVD-based algorithm requires orthogonality of the mode shapes and offers only approximate, not exact recovery.

Recently, atomic norm minimization (ANM) based approaches for line spectrum estimation have been shown to be efficient and powerful for exactly recovering unobserved samples and identifying off-grid frequencies in both single measurement vector (SMV) and multiple measurement vector (MMV) scenarios. In particular, theoretical guarantees have been established for random sampling in time when the sampling times are synchronous and asynchronous across the sensors. However, these guarantees assume randomness of the mode shapes, which is not physically plausible. Moreover, these guarantees suggest that sample complexity per sensor will increase as the number of sensors increases, which is both undesirable and contrary to intuition.

In this work, we consider the modal analysis problem when data is compressed at each sensor via multiplication by a random matrix. We show that ANM can perfectly recover modal parameters even when the mode shapes are not orthogonal. We provide new theoretical analysis on the sample complexity of this scheme. In particular, our theory does not require randomness of the mode shapes, and it shows that the sample complexity per sensor will decrease as the number of sensors increases. Our theory can be interpreted as an extension of the SMV scenario to the MMV scenario.

**B6 - Diffusion-limited mixing by incompressible flows**

Christopher J. Miles, Charles R. Doering, University of Michigan, Ann Arbor.

Incompressible fluid flows can be effective mixers by appropriately spreading a passive tracer throughout a domain. In addition, diffusion is generally perceived as beneficial to mixing due to its ability to homogenize a passive tracer. However, we provided numerical evidence that, in the case where advection and diffusion are both actively present, diffusion produces nearly neutral or even negative effects by limiting the mixing effectiveness of incompressible optimal flows. This limitation appears to be due to the presence of a limiting length scale given by a generalized Batchelor length. This length scale limitation in turn affects long-term mixing rates. More specifically, we consider local-in-time flow optimization under energy and enstrophy flow constraints with the objective of maximizing mixing rate performance. We observe that, for enstrophy-bounded optimal flows, the strength of diffusion has no impact on the long-term mixing rate performance. For energy-constrained optimal flows, however, an increase in the strength of diffusion decreases the long-term mixing rate. We provide analytical lower bounds on mixing rates and length scales achievable under related constraints (point-wise bounded speed and rate-of-strain) by extending the work of Z. Lin et al. (2011) and C.-C. Poon (1996).

**B7 - Ranking with dynamical systems**

W. Garrett Mitchener, College of Charleston.

In data science, a ranking problem is to produce a linear ordering of a set of items that is as consistent as possible with data that includes pairwise comparisons between some of the items, and may include noise and contradictory information. The linear ordering predicts the outcomes of future comparisons between the items. Finding the absolute best ordering is an NP-hard problem.

I will present analyses of dynamical systems built from ranking problems. Each item \( v \) is represented by a particle at \( x_v(t) \in \mathbb{R} \) in a potential well. Each given comparison \( u \lessdot v \) contributes additional terms to the potential that cause forces pushing \( x_u \) to the left and \( x_v \) to the right. The strength of those forces can depend on additional information in the data. When using sports data, for example, \( u \lessdot v \) means that team \( u \) lost to team \( v \), and the strength of the force could be a function of the point spread. The resulting dynamical system conserves energy, so the particles remain in motion indefinitely. Damping terms can be added so that energy leaks from the system, which causes the particles settle to a stable rest state at a local minimum of the energy. The rest state yields a linear ordering of the items. To increase the chances of finding better orderings, trajectories starting from many initial conditions can be computed from the same comparison data. In preliminary benchmarks, the resulting orderings score reasonably well, especially given how quickly trajectories can be computed.

It is natural to apply this dynamic ranking method to time-dependent comparison data, which is an advantage over geometric and combinatorial ranking methods. For example, comparisons can include a time-dependent strength, which allows them to be incorporated sequentially into rankings, or to expire.

Furthermore, the dynamical behavior itself is interesting. Time-dependent comparison data can generate hysteresis and other complex behavior. With no damping, the particles naturally oscillate while exerting asymmetric forces on each other, yielding a network of coupled oscillators different from the usual Kuramoto model.

**B8 - The Role of inertia in the cascading-failure dynamics of power systems**

Samantha Molnar, Elizabeth Bradley, University of Colorado Boulder; Kenny Gruchalla, Bri-Mathias Hodge, National Renewable Energy Lab.

The widespread addition of renewables is transforming power systems in a number of ways. The intermittency in renewable generation sources is a key issue in the dynamics of these systems; equally important—and far less well studied—is the impact of their different inertial properties. Traditional generation sources, such as coal or gas, spin magnets as part of the generation process which means they inherently provide inertia by changing how fast they spin. However, renewable generation
sources do not have this property unless we add power electronic technologies such as smart inverters which create virtual inertia. Since modern power systems contain a mix of traditional and renewable sources, the interplay of the different dynamics can be quite complicated—especially during cascading failures ("blackouts"), in which inertia can play a crucial role. A critical question, then, is whether the risk of cascading failures increases with the penetration of renewable generation sources into the grid. We use techniques from information theory and networks to provide a first answer to this question. We focus on a number of specific "contingencies" (power lines that fail simultaneously), exploring the difference in dynamics between a traditional system and one with increased renewable sources. We use transfer entropy to quantify the pairwise interactions between the different generators and study network properties to determine the contribution of the network topology.

B9 - Fundamental Cupolets of Chaotic Systems

Matthew Morena, Christopher Newport University; Kevin M. Short, University of New Hampshire.

Cupolets represent highly accurate approximations to the unstable periodic orbits of chaotic systems and large numbers can be efficiently generated via a particular control method. Cupolets exhibit the interesting property that a given set of controls will uniquely identify a cupolet regardless of the initial state of the parent chaotic system. We have previously demonstrated that this property allows for controlled transitions between nearly any two cupolets. Now, we discuss how this result can be used to classify cupolets according to their reducibility: a cupolet is classified as fundamental if its orbit cannot be decomposed into the orbits of simpler cupolets and is called composite when a decomposition is possible. Our work introduces a new way to generate higher order cupolets simply by amalgamating fundamental cupolets via sequences of controlled transitions. This allows for large collections of cupolets to be collapsed onto smaller subsets of fundamental cupolets without losing any dynamical information. We also discuss the potential for analyzing a first answer to this question. We focus on a number of specific "contingencies" (power lines that fail simultaneously), exploring the difference in dynamics between a traditional system and one with increased renewable sources. We use transfer entropy to quantify the pairwise interactions between the different generators and study network properties to determine the contribution of the network topology.

B10 - Icicle ripples: how to include impurities?

Stephen Morris, John Ladan, University of Toronto.

Icicles observed in nature and the laboratory often exhibit ribs or ripples with a wavelength close to 1cm around their circumference. Previous experiments on laboratory-grown icicles have shown that the existence of these ripples depends on the presence of (very small) concentrations of impurities in the feed water. However, all existing theoretical models of the icicle ripple instability have ignored the purity of the water.

We present a model of solid icicle growth incorporating the effects of impurities on the freezing point. This model is based on previous work that assumed a thin-film flow over solid ice. We introduce realistic, physically derived boundary conditions for both heat transfer and impurity concentration. A linear stability analysis of this model was performed numerically to high orders of ripple wavenumber.

We show that this more physically complete model of solid icicle growth cannot account for the 1cm wavelength of the ripple instability, because the effects of impurities are inherently too weakly coupled to the freezing dynamics. This suggests that a more complex model must be used, possibly one involving so-called “spongy” ice. Models of the freezing and growth of spongy ice are more strongly affected by impurities in the water. We propose experiments to look for the presence of spongy ice in laboratory-grown icicles in order to test this hypothesis.

B11 - Experimental Study of Parallel-Connected DC-DC Buck-Boost Converters FPGA Chaos Controlled

Rachel Lee, Ammar N. Natsheh, Higher Colleges of Technology, Dubai Women’s College.

Chaos control is used to design a controller that is able to eliminate the chaotic behavior of nonlinear dynamic systems that experience such phenomena. This paper discusses the use of the FPGA micro-controller as a controller of a parallel-connected DC-DC buck-boost converter, the goal of this paper is to build a controller that is capable of controlling the output current of a photovoltaic cell and minimize the effect of the module buck-boost converter chaotic behavior on the output voltage. To achieve this goal this paper presents two different methods, FPGA control the duty cycle and the frequency of the output controlling signal, this technique is done experimentally through software (FPGA code) and a delayed feedback controller hardware in a module converter in the continuous-current conduction mode (CCM). Thus, this paper shows the FPGA capabilities in the power industry and it’s specifies a guideline to overcome some of the obstacles when dealing with an FPGA as a buck-boost converter controller in renewable energy systems.

B12 - Slow manifolds in the aerodynamic descent of animals and plants

Gary K. Nave, Shane D. Ross, Virginia Tech.

A large variety of animals exhibit the capacity for gliding flight, including but not limited to species of squirrels, lizards, fish, ants, and snakes. Historically, to understand these animals, researchers have considered equilibrium glide angle, defined by the mean lift-to-drag ratio. However, in a glide, most of the animal’s motion occurs away from equilibrium. To more accurately understand the glide, we consider a 2 degree-of-freedom model, which considers lift and drag coefficients as functions of angle of attack and the animal’s pitch angle with respect to the ground as a control parameter. In this model, we find that most of the dynamics occur along a slow manifold in the presence of a fast, super-stable direction. Because all trajectories collapse onto this slow manifold, we may consider it as a higher-dimensional analog for terminal velocity, which is itself a codimension-one structure in a 1 degree-of-
freedom model. In this work, we present a variety of methods for calculating this terminal velocity manifold, which give new insight to its identity. We also prescribe the pitch dynamics in extended phase space and re-consider gliding and fluttering dynamics as occurring on a two-dimensional terminal velocity manifold embedded in a three-dimensional system. These phase space structures may be leveraged for efficient control strategies of engineered aerial dispersal systems in the future.

**B13 - The network counterpart of the butterfly effect**

Takashi Nishikawa, Adilson E. Motter, Northwestern University; Jie Sun, Clarkson University.

The relation between network structure and dynamics is determinant for the behavior of complex systems in numerous domains. An important long-standing problem concerns the properties of the networks that optimize the dynamics with respect to a given performance measure. Here, we show that such optimization can lead to sensitive dependence of the dynamics on the structure of the network. Specifically, using diffusively coupled systems as examples, we demonstrate that the stability of a dynamical state can exhibit sensitivity to unweighted structural perturbations (i.e., link removals and node additions) for undirected optimal networks and to weighted perturbations (i.e., small changes in link weights) for directed optimal networks. As mechanisms underlying this sensitivity, we identify discontinuous transitions occurring in the complement of undirected optimal networks and the prevalence of eigenvector degeneracy in directed optimal networks. These findings establish a unified characterization of networks optimized for dynamical stability, which we illustrate using Turing instability in activator-inhibitor systems, synchronization in power-grid networks, network diffusion, and several other network processes. Our results suggest that the network structure of a complex system operating near an optimum can potentially be fine-tuned for a significantly enhanced stability compared to what one might expect from simple extrapolation. On the other hand, they also suggest constraints on how close to the optimum the system can be in practice. Finally, the results have potential implications for biophysical networks, which have evolved under the competing pressures of optimizing fitness while remaining robust against perturbations. Reference: T. Nishikawa, J. Sun, and A. E. Motter, Sensitive Dependence of Optimal Network Dynamics on Network Structure, Phys. Rev. X 7, 041044 (2017).

**B14 - Adaptable Susceptibility in Epidemiological Models in Networks**

Renato Pagliara, Naomi E. Leonard, Princeton University.

The spread of infectious diseases has been a long-standing area of research across a diverse range of fields including biology, physics, computer science, engineering, economics, and other social sciences. Models drive the study of these processes because, in most cases, datasets are scarce and experiments are not viable. Spreading processes in epidemiology are usually described by compartmental models where the population is divided into groups and equations define the flow of individuals between these groups.

Much of the work on epidemiological models has been dedicated to the Susceptible-Infected (SI), Susceptible-Infected-Susceptible (SIS), and Susceptible-Infected-Recovered (SIR) classes of models. In the SI model, susceptible individuals become infected through contact with already infected individuals. The SIS model allows infected individuals to become susceptible again after recovering from the infection. In the SIR model, recovered individuals acquire immunity to the infection and cannot become infected again. One of the main results in classical epidemiology is that an epidemic takes place only if the basic reproduction number $R_0$ is larger than one, where $R_0$ represents the average number of secondary infections that a single infected individual generates.

Here we study a model with reinfection called the Susceptible-Infected-Recovered (SIRI). We show that the SIRI model specializes to the SI, SIS, and SIR models as the rate of infection after recovery is varied from zero to infinity.

Unlike the SI, SIS, and SIR models, the SIRI model can have an epidemic even when $R_0$ is less than one. We find conditions for bistability in which a small difference in the initial number of infected individuals is sufficient to change the long term behavior of the system from a disease-free steady-state to an endemic one where the fraction of infected individuals is nonzero.

We then expand these results to arbitrary interaction network topologies where each node can be susceptible, infected, or recovered. We model the dynamics on the network through the N-intertwined mean-field approximation and consider two different cases. In the first case we let all individuals have the same rate of infection and the same rate of recovery, while in the second case we allow each individual to have a different rate of infection and a different rate of recovery. In each case, we find the equilibria, stability properties, and threshold conditions for epidemics.

Our results generalize the results for the SI, SIS, and SIR models both in compartmental and network settings. Although our work focuses on the context of epidemics, the same results are directly applicable to many different spreading processes in which individuals are more or less likely to become infected again after recovering, such as the spread of computer viruses, the propagation of faults and failures, and the spread of news and rumors in social networks.
B15 - Stability of Double Pulse Solutions to the 5th order KdV Equation: A Numerical Approach

Ross Parker, Bjorn Sandstede, Brown University.

The fifth-order Korteweg-de Vries equation (KdV5) is a nonlinear partial differential equation used to model dispersive phenomena such as plasma waves and capillary-gravity water waves. For wave speeds exceeding a critical threshold, KdV5 admits a countable family of double-pulse traveling-wave solutions, where the two pulses are separated by a phase parameter multiplied by an integer N. It is known that the double pulses are unstable for even N, and it has also been shown that these pulses have either a quadruplet of eigenvalues or a pair of purely imaginary eigenvalues near the origin when N is odd. Moreover, the latter case arises provided the associated eigenfunctions are square-integrable. It is not known which of these two cases arises in KdV5. We provide the results of extensive numerical computations that indicate that the eigenvalues are indeed purely imaginary. We also present numerical Krein matrix computation for periodic waves that indicate that long-wavelength double-pulse wave trains are stable.

B16 - Using Machine Learning to Replicate Chaotic Attractors and Calculate Lyapunov Exponents from Data

Jaideep S. Pathak, Brian Hunt, Michelle Girvan, Ed Ott, University of Maryland, College Park; Zhixin Lu, University of Pennsylvania.

Networks of nonlinearly interacting neuron-like units have the capacity to approximately reproduce the dynamical behavior of a wide variety of dynamical systems. We demonstrate the use of such artificial neural networks for reconstruction of chaotic attractors from limited time series data using a machine learning technique known as reservoir computing. The orbits of the reconstructed attractor can be used to obtain approximate estimates of the ergodic properties of the original system. As a specific example, we focus on the task of determining the Lyapunov exponents of a system from limited time series data. Using the example of the Kuramoto-Sivashinsky system, we show that this technique offers an attractive method for estimating a tem from limited time series data. Using the example of the Kuramoto-Sivashinsky example, we focus on the task of determining the Lyapunov exponents of a sys-

approximate estimates of the ergodic properties of the original system. As a specific
reservoir computing. The orbits of the reconstructed attractor can be used to obtain
attractors from limited time series data using a machine learning technique known as
numerical Krein matrix computation for periodic waves that indicate that

demonstrate the use of such artificial neural networks for reconstruction of chaotic
mately reproduce the dynamical behavior of a wide variety of dynamical systems. We
for application of chaos in practical systems, because parameters often cannot be
set with enough accuracy and may vary while in operation. It has been shown that
robust chaos is possible in 1D and 2D piecewise smooth systems, and the existence
and stability conditions of robust chaos in piecewise linear normal form maps were
derived in [1, 2]. However, the issue of robust chaos in 3-D piecewise smooth systems
has not been address so far. In this work, we address this question with reference
to the 3-D piecewise linear normal form map:

\[ X_{n+1} = F_\mu(X_n) = \begin{cases} A_l X_n + \mu C, & \text{if } x_n \leq 0 \\ A_r X_n + \mu C, & \text{if } x_n \geq 0 \end{cases} \]

where \( X_n = (x_n, y_n, z_n)^T \in \mathbb{R}^3 \), \( C = (1, 0, 0)^T \in \mathbb{R}^3 \), \( A_l \) and \( A_r \) are real valued
3 \times 3 matrices

\[
A_l = \begin{pmatrix} \tau_l & 1 & 0 \\ -\sigma_l & 0 & 1 \\ \delta_l & 0 & 0 \end{pmatrix} \quad \text{and} \quad A_r = \begin{pmatrix} -\sigma_r & 1 & 0 \\ \tau_r & 0 & 1 \\ \delta_r & 0 & 0 \end{pmatrix}
\]

Let \( U_L \) and \( S_L \) be the unstable and stable manifold of \( L^* \) and \( U_R \) and \( S_R \) be the
unstable and stable manifold of \( R^* \) respectively. Note that the unstable manifolds
are one-dimensional and the stable manifolds are two-dimensional. All intersections
of the unstable manifolds with the \( x = 0 \) map to \( z = 0 \). Since one linear map
changes to another linear map across the \( x = 0 \) line, \( U_L \) and \( U_R \) experience fold
along the \( x \) axis and all images of the fold points are also fold points. By a similar
argument, we conclude that \( S_L \) and \( S_R \) fold along the \( z = 0 \) plane and all the
pre-images of fold points are fold points.

We demonstrate the existence of robust chaos in this system by plotting the
Lyapunov exponents. We show that robust chaos occurs as a result of the homoclinic
connection between the stable \((S_R)\) and unstable manifold \((U_R)\), and the uniqueness
of the attractor is proved by the heteroclinic connection between the stable \((S_L)\)
and unstable \((U_R)\) manifold. Stable manifold \((S_L)\) acts as a basin boundary, and
when the attractor touches the basin boundary a boundary crisis occurs and the orbit
loses its stability. We have analytically calculated the condition for occurrence of
homoclinic connection, heteroclinic connection and boundary crisis. Thus we have
calculated the existence and stability conditions of robust chaos in 3-D piecewise
linear normal form map.

References

B18 - Optimal switching between geocentric and egocentric navigational strategies in varying environments

Orit Peleg, Jeffrey Lipnick, Elizabeth Bradley, University of Colorado at Boulder.

Animals use a combination of egocentric navigation driven by the internal integration of environmental cues, interspersed with geocentric course correction and reorientation. These processes are accompanied by uncertainty in sensory acquisition of information, planning and execution. Inspired by observations of dung beetle navigational strategies that show switching between geocentric and egocentric strategies, we consider the question of optimal reorientation rates for the navigation of an agent moving along a preferred direction in the presence of multiple sources of noise. We recently addressed this using a model that takes the form of a correlated random walk at short time scales that is punctuated by reorientation events leading to a biased random walks at long time scales (Peleg and Mahadevan, R. Soc. Open Sci., 2016). This allows us to identify optimal alternation schemes and characterize their robustness in the context of noisy sensory acquisition as well as performance errors linked with agent-environment interactions. The original model assumes that environmental conditions (e.g., the roughness of the surface) are constant, while in reality, animals are more likely to experience varying environments. Here we consider an extension of the model where the agent is navigating on mixed roughnesses that alternates back and forth from flat to rough. We show that it is not straightforward to interpolate the optimal behavior on a mixed roughness based on the known optimal behavior on a homogenous roughness, as their relation is non-linear. This opens up an avenue to study optimal sensory and adaptive decision making in the context of mixed roughnesses, which we are exploring via Reinforcement Learning methods.

B19 - Symbolic tools for the study of Homoclinic chaos

Krishna Pusuluri, Andrey Shilnikov, Georgia State University.

We developed a novel GPU based computational approach based on the symbolic encoding of homoclinic bifurcations occurring in Lorenz-like systems. This approach can reveal the wealth of underlying structures of complexity such as homoclinic and heteroclinic connections, Bykov T-point spirals, saddles, and inclination flip points. It can also unfold the long term behavior of these systems to identify regions of simple Morse-Smale dynamics of stable equilibria and periodic orbits, or regions of structurally unstable chaotic dynamics. Previously existing techniques such as parametric continuation or Lyapunov exponents only accomplish a small fraction of such dynamical details.

B20 - Asymptotic dynamics in networks of complex quadratic maps

Anca R. Radulescu, Simone Evans, SUNY New Paltz.

Behavior under iterations of complex quadratic maps has been one of the earliest and most studied topics in discrete dynamics, since the dawn of the 19th century. However, subtler aspects related to coupled dynamics of logistic maps remain largely unexplored. We study ensemble dynamics of quadratic maps, organized as nodes in a self-interacting network.

We investigate how the traditional theory of Fatou and Julia changes in these cases, illustrating how the hardwired structure (adjacency graph and coupling weights) can affect dynamics (captured by the asymptotic behavior of the critical orbits). While some results extend directly from the single map case, some traditional theorems no longer hold for networks. We will show how the hyperbolic bulbs of the Mandelbrot are perturbed by introducing coupling. We will give examples of simple networks in which the Mandelbrot set is no longer connected. We will discuss in which way the connectedness locus for Julia slices still relates to the network Mandelbrot set.

While not immediately applicable to the life sciences, studying dynamics in simple model networks may increase our basic understanding of the effects of hardwiring on ensemble dynamics. We discuss how this may be a stepping stone towards understanding processes such as systemic learning, organization and restructuring in functional networks.

B21 - Self-similar solutions of diverging slip rate on faults with heterogeneous friction

Sohom Ray, Robert C. Viesca, Tufts University.

Frictional sliding of an extended system coupled with its deformation is ubiquitous. One of them is a geological fault, an interface within a continuum across which relative displacement (slip along the interface) may occur. A slip distribution instantaneously leads to quasistatic changes in shear traction on the fault, which adds to the traction due to any external forcing. Slip occurs when and where fault’s shear stress equals its frictional strength.

Rock friction experiments suggest that, contrary to the popular belief of constant friction, a fault’s frictional strength depends on instantaneous slip rate and its state (or history). Such a rate-and state-dependent frictional strength has a direct and subsequent evolutionary response towards (logarithmic) changes in slip velocity, magnitudes of which are controlled by parameters $a$ and $b$ respectively [e.g., Ruina 1983]. Those frictional properties dictate whether a slip diffuses on the fault (if $a > b$) or diverge within a compact support (if $a < b$, our focus here). Latter is ensured if the wavelength of perturbations to steady-state sliding exceeds a critical lengthscale.

Governing evolution equations, being invariant under time translation and amplitude scaling, suggests that that the blow-up might occur as $v(x,t) = W(x,t)/t_f$, where $W(x,t)$ is the slip rate on the fault, and $t_f$ is the slip rate at which the fault is in steady-state sliding. Our main concern is to understand the robustness of this blow up under perturbations of the frictional properties. To this end, we employ the bifurcation theory of the Dullemond and Meijer [2002] model that describes the evolution of slip rate on a fault when its frictional properties are perturbed.
where \( t_f = (t_\alpha - t) \) is the time to instability. The diverging slip rate approaches a self similar profile, if \( t_f \partial W/\partial t \) attains a steady state and hence can be obtained using \( t_f \partial W/\partial t = 0 \) a priori.

Further, setting \( t_f \partial W/\partial t = 0 \) a priori could lead to multiple self similar solutions in situations where friction parameters are non-uniformly distributed [Ray and Viesca, 2017]. However, only subset of them are attainable for which perturbation grows at a rate \( \sim o(t_f^{-1}) \) as \( t_f \to 0 \). Further, the length scales over which frictional properties vary over the fault could alter the attainability of the self similar solutions. We develop a low parameter model with \( \alpha \) and \( \kappa \) respectively controlling the magnitude and length scale of heterogeneity. Similarity solutions at specific regions on fault gain or lose stability (as pitchfork and transcritical bifurcations) as \( \alpha \) and/or \( \kappa \) is varied. We conclude that only specific regions on faults an unstable slip could nucleate. Such preferred earthquake nucleation locations are otherwise lost when frictional heterogeneities are avoided for model simplification purposes.

**B22 - Modulation of spike-frequency adaptation and learning rules optimize network function across sleep-wake states.**


The brain controls its functional state by releasing molecules that control the biophysical properties of neurons and modulate the strength of connections between them. The effects of these neuromodulators lead to divergent network dynamics that result of an interaction between network structure and cellular excitability properties. A prominent neurochemical marker of arousal state are the levels of acetylcholine (ACh) and norepinephrine (NE) release within the brain. The highest level of ACh tone is seen during REM sleep and during periods of high vigilance. During NREM sleep ACh level is at its lowest. While awake, but in a resting state, ACh release is at a moderate level. NE is released at its highest levels during waking and at similar low levels during sleep, regardless of stage. Here we show that dynamic changes in ACh levels that are associated with arousal state control informational processing functions of networks though its effects on the degree of spike-frequency adaptation (SFA; an activity dependent decrease in excitability) displayed by single cells. On the other hand, NE controls the polarity of learning through spike-timing dependent plasticity (STDP); favoring synaptic strengthening during waking and while being more balanced during sleep. Using a simple biophysical model of a cortical network which includes a slow potassium M-current and learns to encode input through STDP we show that decreases of cholinergic tone (which increases SFA) shifts the network activity from a stable high frequency pattern to a traveling wave of activity like that of SWS. The high ACh and NE (waking) stable activity pattern can be localized to regions of the network with enhanced recurrent excitatory synapses (spatial attractors) and are highly sensitive to even slight increases in synaptic strength. During this period STDP biased toward strengthening reinforces spatial attractors and can encode novel attractors in response to changing external input. In NREM the transition from stable to traveling dynamics occurs in conjunction with a decrease in sensitivity to spatial attractors, but allows for paths between them to be encoded via STDP. Finally, during REM stationary dynamics and selectivity for attractors returns, but the balanced STDP rule leads to destabilization of attractors. This leads to activity that is transiently stable then follows the paths formed during NREM. Importantly, we show that modulation of a cellular properties and STDP can regulate the informational processing state of a network. We propose that the ACh and NE levels associated with arousal states lead to a cooperative process during waking, NREM, and REM where waking is optimized for encoding of new information or the stable representation of relevant memories, NREM is optimized for encoding connections between currently stored memories, and REM is optimized for reinforcing connections between attractors and consolidating stored memories. This work provides a mechanistic insight into the role of dynamic control of neuronal properties in the encoding, consolidation, and maintenance of memories within the brain.

**B23 - Chaotic advection in active nematics**

**Eric Roberts, Amanda Tan, Linda Hirst, Kevin Mitchell, University of California, Merced.**

The recent surge of research into active materials is an exciting development in soft matter physics. Unlike traditionally studied fluids, active fluids are not in equilibrium. Instead, they continuously consume energy to generate internal motion, which can subsequently produce large-scale flows and rich emergent dynamical structures, such as moving topological defects. These moving defects can wind around one another to generate chaotic mixing. We report here on experimental and theoretical work on a biologically inspired active nematic liquid crystal. Densely packed microtubules slide antiparallel to each other at a controlled rate due to kinesin molecular motors. The resulting chaotic advection is studied experimentally using the tools of particle tracking, particle image velocimetry, and fluorescence imaging of labeled tracers. Experimental data are analyzed and interpreted in the context of topological dynamics, thereby bridging the fields of chaotic advection and active fluids. We focus on the topological entropy, measured from the braiding of tracer trajectories and on the local Lyapunov exponent, measured from the divergence of neighboring tracers.

**B24 - Spacetime symbolic dynamics and local presentations of 1+1D sofic shifts**

Adam Rupe, James P. Crutchfield, UC Davis.

Epsilon machines are probabilistic versions of the future cover (right Krieger cover) of one-dimensional shift spaces. The states of the epsilon machine, causal states, are minimal sufficient statistics of the past for optimal prediction of the future of a stationary stochastic process. Generalizing to spatiotemporal systems, the local
causal states are minimal sufficient statistics of local pasts for optimal prediction of local futures, where local pasts and futures are given as lightcones. The local causal states together with the induced transition structure over them define a "spatially-local epsilon machine". Unlike in one dimension however, the topological variant of this construction does not form a consistent presentation of the underlying shift space. We introduce a new local presentation, the V-machine, that consistently captures the allowed spacetime blocks of the shift. This presentation's states are still minimal sufficient statistics of past lightcones for optimal prediction of local futures. However, the choice of local future is modified, following Weiss's original semi-group construction of sofic shifts. We give several examples from spacetime fields of elementary cellular automata and other related 2D shifts. Our preliminary results show that while V-machines clean up several of the mathematical loose ends of local epsilon machines, they also lose the utility of the local causal states, specifically when it comes to uncovering hidden symmetries of a spacetime shift. This poses a challenge to optimally predicting spatiotemporal dynamical systems.

**B25 - Investigation of Period-3 Orbits in the Malkus-Lorenz Waterwheel**

George Rutherford, Duncan Ooms, Noah Osman, Richard F. Martin, Jr., Department of Physics, Illinois State University.

The Malkus-Lorenz waterwheel is a familiar demonstration device for nonlinear dynamics as a mechanical analog of the Lorenz equations. Careful experimental results have only been reported in the last few years, however, revealing a rich dynamical system with significant departures from the predictions of numerical solution of the ideal model equations. Our apparatus has 36 acrylic cylinders around the periphery of a wheel with a 9 in (0.229 m) radius. The wheel’s shaft is held by two air bushings and a flat air bearing to support the axial force, reducing friction to its practical minimum. An aluminum ring around the periphery of the wheel passes between the poles of variable gap magnets, producing a frictional torque approximately proportional to the angular velocity. The wheel’s angular position is measured between the poles of variable gap magnets, producing a frictional torque approximately proportional to the angular velocity. The wheel’s angular position is measured by differentiation via fast Fourier transformation. This poster will report investigation of period-3 orbits near the transition to chaos using the strength of the magnetic brake as the bifurcation parameter. Preliminary data indicate that a small period-3 window exists in that region that is not seen in numerical solutions of the ideal model equations. However, we are developing a more realistic computer model, and preliminary data from that model do show a small period-3 window. Analysis of the experimental period-3 orbits show no evidence of higher periodicity, and first return maps as well as detailed analysis of the phase portraits confirm that the orbits are period-3. Their nearness to the cascading transition to chaos is puzzling, however. In addition, we will report attempts to more fully characterize the velocity-dependent braking torque, which is not strictly linear.

**B26 - Gender differences in hepatic one-carbon metabolism**

Farrah Sadre-Marandi, Mike Reed, Fred Nijhout, Mathematical Biosciences Institute.

It is known that there are gender differences in one-carbon metabolism (OCM). Women in the child-bearing years exhibit lower plasma homocysteine, higher betaine and choline, and lower S-adenosylmethionine. Various enzymes in OCM are up-regulated or downregulated in women due to estrogen. Insulin and glucose affect some enzymes of OCM and change furthermore during pregnancy. All of these results suggest that a mechanistic understanding of how enzymatic differences in women affect OCM is important for precision medicine.

The reaction diagram for the folate and methionine cycles in OCM is very complicated consisting of loops within loops. Furthermore, many substrates in the network influence, through allosteric binding, the activity level of enzymes at distant locations in the network. A mathematical model of folate and methionine metabolism is used to study the enzymatic changes in women of child-bearing age and the resulting concentrations of metabolites. In each case the results are compared to clinical and experimental studies. The causal mechanisms by which the gene expression or enzyme activity changes in women that lead to the metabolite changes will be discussed.

**B27 - Symmetries of states of a ring of nanoelectromechanical oscillators**

Anastasiya Salova, Jeffrey Emenheiser, UC Davis; Raissa D’Souza, UC Davis, SFI.

Systems of nanoelectromechanical oscillators (NEMS) are a useful tool to study dynamical properties of complex multistable networks since they make it possible to combine theory, simulation, and experiment to gain insight into nonlinear phase-amplitude oscillators. We study systems of 3 and 8 such oscillators arranged in a uniform ring topology with nearest neighbor coupling at different parameter regimes. Those topologies correspond to currently available NEMS experimental setups. We also consider rings of 6 oscillators to illustrate all types of symmetry breaking in our system. In addition to the fully synchronized state and splay states at unit amplitude, the analytic form and stability properties of which are well-understood, we observe states at non-uniform amplitude that are harder to analyze. We classify these states based on their symmetry group and compare the set and fraction of stable states obtained from experiment and theory at the same parameter values. Our findings are an important step towards exploring control strategies for symmetric amplitude-phase oscillator networks using amplitude noise injections and phase kicks. To illustrate that, we present some examples of noise-induced and kick-induced attractor switching in both simulation and experiment.
An operator $T$ on a Banach space $X$ is called hypercyclic if there is a vector $x \in X$ (called a hypercyclic vector) whose orbit $\{T^n x, n = 1, 2, 3, \ldots\}$ is dense in $X$. The orbits of hypercyclic vectors are topologically transitive. We construct hypercyclic vectors for the iterated differential operator $D$ that are chaotic, that is, in addition to topological transitivity, there exist dense periodic orbits of all periods that are approached arbitrarily closely by $D^n x$. We then suspend the orbit $D^n x$ to a flow by substituting the fractional differential operator $D^{\eta}$, where $\eta$ is a continuous real parameter. The orbit of $D^{\eta} x$ is then a continuous curve that is dense in $X$. The orbit of $D^{\eta} x$ is called a hypercyclic vector whose orbit
for human mobility), optics (exploration of the use of nonlinear Hamiltonian optics in communication and detection), uses of fluid patterns and dynamical structures (novel methods of pattern visualization and cyclonic and dielectrophoretic particle separation), and biomedicine (modeling kinetic networks for disease detection and making circuit analogs of neuronal structures for prosthetic and robotics applications). A common base (the crust of the pie) consists of analytical, computational, and experimental methods broadly used in nonlinear dynamics research. The program is part of a larger NSF-supported effort to show students how to connect Physics with Innovation and Entrepreneurship (hence PIE). However, we like the culinary metaphor: nonlinear dynamics is a powerful domain for making a bigger pie whose slices let students advance their careers through genuine contributions to human welfare.

B34 - The Fluid Dynamics of the Brain’s Waste Removal System

Jeffrey Tithof, John H. Thomas, Douglas H. Kelley, University of Rochester; Humberto Mestre, Maiken Nedergaard, University of Rochester Medical Center.

The glymphatic system, discovered in 2012, is a pathway in which cerebrospinal fluid (CSF) flows through the brain to remove waste. CSF, which is a clear water-like fluid, enters the brain along spaces surrounding arteries, flows through the brain tissue to sweep away waste, then exits along spaces surrounding veins. A growing body of evidence suggests that a breakdown in healthy glymphatic flow can lead to neurodegenerative diseases like Alzheimer’s disease, which is characterized by a build-up of protein waste in the brain. Recent advancements in imaging have allowed for direct observation of CSF flow in living mouse brains. By injecting fluorescent microspheres into the mouse’s brain, recording sequential images of their motion, and performing particle tracking velocimetry, we have obtained the first ever quantitative measurements of CSF flow. We observe that the flow has substantial spatial variations in speed, with stagnation points near arterial bifurcations, where waste is known to accumulate. We also observe that the flow is pulsatile. To test which physiological factors affect the dynamics of the flow, we record simultaneous measurements of the mouse’s vital signs (heart beat, respiration, etc.). Interestingly, we observe that the pulsatility of the flow is synchronized to the heart beat, even though the heart itself does not pump CSF. Our results suggest that each heart beat generates a traveling wave in the arterial wall which drives CSF flow via peristalsis. We hypothesize that this waveform is optimal for peristaltic pumping and discuss mechanisms which may change this waveform, potentially leading to lessened waste removal in the brain and development of neurodegenerative disease.

B35 - Modeling the dynamics of liver regeneration

Babita K. Verma, S. Pushpavanam, Indian Institute of Technology Madras, Chennai, India; Rajanikanth Vadigepalli, Thomas Jefferson University.

Among all the organs, liver has the unique ability to regenerate completely even after most of its mass is lost. This process of regeneration takes place via a unique mechanism in which differentiated hepatocytes reenter cell cycle to replenish the lost cells. While liver regeneration is robust in healthy individuals, chronic disease diminishes this regenerative capacity making liver surgery less than effective, and can even lead to liver failure in patients. This is a matter of concern, since majority of patients recover partially [1], which can lead to liver failure for even slightly higher level of resection. Various factors are responsible for partial recovery or liver failure such as the remnant liver mass, preexisting liver disease, age, obesity, surgery related factors such as excessive bleeding leading to ischemia, and cell death. What controls the different modes of regeneration most significantly remains unanswered.

In this work, we used dynamical systems approach to identify the key parameters that control the liver recovery or failure, and studied their effect on the threshold of liver failure via phase portrait technique. We employed a recently developed mathematical model [2] to account for different regeneration modes such as suppressed, normal and liver failure based on the extent of liver recovery. Using a control systems approach, we identified two sensitive parameters, metabolic demand and cell death, that are responsible for these regeneration modes. To comprehensively understand the role of these sensitive parameters we performed Sobol sampling analysis to delineate the parameter subspaces that corresponded to three response modes - full recovery, suppressed recovery, failure. We noticed that the subspace of the full recovery is enveloped by the subspace of the suppressed growth.

We then employed a dynamical systems approach of phase plane to study the effect of different levels of resection on the mass recovery. We observed that the regeneration system exhibits multiple steady states; two of the stable steady states correspond to liver recovery and liver failure. We utilized phase plane analysis to compute the threshold of resection that separates the recovery versus failure response modes as the two basins of attraction. Our simulations demonstrated that the basin of attraction for recovery shrinks and finally vanishes as a function of the metabolic demand and cell death leading to a change in the multiplicity of the system. Varying these sensitive parameters resulted in a phase plane with either one sink of failure or two sinks of recovery and failure. This change in the multiplicity of the system has clinical implications, such that in some cases, liver surgery may not be a viable option for treatment, depending on the patient-specific metabolic demand and cell death parameters.

Dr. Christopher M. Kribs, University of Texas at Arlington, and Sara Aton, University of Michigan, Ann Arbor.

We validate the model's prediction with historical Democratic and Republican party positions estimated from congressional voting records, and find strong support.

B37 - Why are US parties so polarized? A “satisficing” dynamical model

Vicky Chuqiao Yang, Daniel Abrams, Adilson Motter, Northwestern University; Georgia Kernell, UCLA.

The US Democratic and Republican parties have been polarizing for the past sixty years while little polarization has occurred in the general US public. How is it possible for parties to polarize without the voting population doing the same? Previous work modeling political elections often assumes voters maximize their utility, which is challenged by psychological studies. Here, we propose a possible mechanism of party polarization through a first-principles mathematical model where voters “satisfice:” they do not exhaustively seek out what’s best, but settle for “good enough.” Our model predicts that party polarization can result from parties being less ideologically inclusive. We validate the model’s prediction with historical Democratic and Republican party positions estimated from congressional voting records, and find strong support.

B38 - Topological methods for analyzing two dimensional flows

Tomoo Yokoyama, Kyoto University of Education; We introduce new topological methods, which are word representations and graph representation, to analysis two dimensional flows generated by the complex velocity potentials of uniform flows and point vortices. Applying the topological methods to a plate in a time-dependent flow under mild conditions, we can estimate when the lift-to-drag ratios of the plate are maximal and can determine the intermediate topologies of the uniform flow. Moreover, we state the possibilities of analyzing ocean phenomena and medical phenomena by the topological methods. In addition, we discuss the relations between topological structures and data structures. Finally, we discuss low-dimensional dynamical systems which are theoretical backgrounds of the topological methods for analyzing two dimensional flows.

B39 - Fast cheater migration stabilizes coexistence in a public goods dilemma on networks

Glenn S. Young, Andrew Belmonte, Pennsylvania State University.

Cooperation is frequently considered an unsustainable strategy: if an entire population is cooperating, each individual can increase its overall fitness by choosing not to cooperate, thereby still receiving all the benefit of its cooperating neighbors while no longer expending its own energy. Observable cooperation in naturally-occurring public goods games is consequently of great interest, as such systems offer insight into both the emergence and sustainability of cooperation. Here we consider a population that obeys a public goods game on a network of discrete regions (that we call nests), between any two of which individuals are free to migrate. We construct a system of piecewise-smooth ordinary differential equations that couple the within-nest population dynamics and the between-nest migratory dynamics. Through a combination of analytical and numerical methods, we show that if the workers within the population migrate sufficiently fast relative to the cheaters, the network loses stability first through a Hopf bifurcation, then a torus bifurcation, after which one or more nests collapse. Our results indicate that fast moving cheaters can act to stabilize worker-cheater coexistence within network that would otherwise collapse.

B40 - Ebola: Impact of hospital’s admission policy in an overwhelmed scenario

Md Mondal Hasan Zahid, Dr. Christopher M. Kribs, University of Texas at Arlington.

Infectious disease outbreaks sometimes overwhelm healthcare facilities with patients. A very recent case happened in West Africa in 2014 when outbreak of Ebola virus overwhelmed healthcare facilities in Sierra Leone, Guinea and Liberia. In this type of scenario, how many patients can hospitals admit to minimize the burden
of the epidemic? In this work, we tried to find what type of admission policy by a hospital can better serve the community. Our result shows the determination of the policy depends on the initial estimation of the basic reproduction number, $R_0$. When the outbreak grows extremely fast ($R_0 \gg 1$) it is better to stop admitting patients after reaching the carrying capacity because overcrowding in the hospital makes the hospital setting ineffective at containing infection, but when the outbreak grows only a little faster than the system’s ability to contain it ($R_0 \geq 1$), it is better to continue admitting patients beyond the carrying capacity because limited overcrowding still reduces infection more in the community. However, when $R_0 \leq 1.012$, both policies result the same because the number of patients will never go beyond the maximum capacity.

**B41 - Identical synchronization of nonidentical oscillators: when only birds of different feathers flock together**

Yuanzhao Zhang, Adilson E. Motter, Northwestern University.

An outstanding problem in the study of networks of heterogeneous dynamical units concerns the development of rigorous methods to probe the stability of synchronous states when the differences between the units are not small. Here, we address this problem by presenting a generalization of the master stability formalism that can be applied to heterogeneous oscillators with large mismatches. Our approach is based on the simultaneous block diagonalization of the matrix terms in the variational equation, and it leads to dimension reduction that simplifies the original equation significantly. This new formalism allows the systematic investigation of scenarios in which the oscillators need to be nonidentical in order to reach an identical state, where all oscillators are completely synchronized. In the case of networks of identically coupled oscillators, this corresponds to breaking the symmetry of the system as a means to preserve the symmetry of the dynamical state—a recently discovered effect termed asymmetry-induced synchronization (AISync). Our framework enables us to identify communication delay as a new and potentially common mechanism giving rise to AISync, which we demonstrate using networks of delay-coupled Stuart-Landau oscillators. The results also have potential implications for control, as they reveal oscillator heterogeneity as an attribute that may be manipulated to enhance the stability of synchronous states.