Why should dynamicists be interested in data assimilation?
Invited talk. Thursday, 8:45 AM - 9:20 AM
Chris Jones, University of North Carolina.

Dealing with data is arguably the challenge of our times. The aim is to tease as much information as we can from the available data related to a particular physical situation. This can be in the service of making predictions about how that system will behave in the future, or understanding its inner workings. The burgeoning area of data science tends to emphasize the profusion of data and downplay their provenance. But data come from distinct sources, and that matters. There are data from computational runs of models based on underlying physical laws and data from observations in the real world. Both understanding and prediction are significantly enhanced if these are treated as different sources of information. Data assimilation (DA) is the mathematical subject that takes this viewpoint and has the objective of gaining optimal information by combining the data from these sources in intelligent ways. The ideas of dynamical systems have shaped many of the modern techniques of DA and I will argue that the dynamics community has much more to offer.

Observability analysis and estimator design for a cardiac cell model
Contributed talk. Thursday, 9:20 AM - 9:40 AM
Laura Muñoz, Rochester Institute of Technology.
Co-authors: Anthony Guzman, Boston University; Ryan Vogt, University of Minnesota; Claire Charron and Kalyan Pusarla, Rochester Institute of Technology.

Certain cellular variables, such as ionic concentrations and gating states, are thought to be critical to the formation of dangerous cardiac arrhythmias, but some of these quantities may be difficult or impossible to measure directly during in vitro experiments. To help remedy this shortcoming, we examined the Luo-Rudy dynamic (LRd) model, which is a 17th-order nonlinear ODE model of the action-potential dynamics of a cardiac cell, as a basis for reconstructing important cellular variables. To determine whether measurements of any individual dynamical variable were sufficient to estimate the remaining variables, we used a numerical linearization approach to analyze a model property called observability. The observability results indicated that certain measurements, including membrane potential, were sufficient for estimating the other dynamical variables in the model, and that strength of observability tended to increase with increasing heart rate. For selected scenarios, Kalman filters were designed and tested with simulated data to check the effectiveness of reconstructing unmeasured variables from different types of measurements. Observability analysis and state estimation methods, such as Kalman filtering, constitute a promising approach for allowing researchers to gain a more complete understanding of the dynamical behavior of cardiac cells.

Simon’s fundamental rich-gets-richer model entails a dominant first-mover advantage
Invited talk. Thursday, 9:40 AM - 10:15 AM
Peter Dodds, University of Vermont.

Herbert Simon’s classic rich-gets-richer model is one of the simplest empirically supported mechanisms capable of generating heavy-tail size distributions for complex systems. Simon argued analytically that a population of flavored elements growing by either adding a novel element or randomly replicating an existing one would afford a distribution of group sizes with a power-law tail. In this talk, I will show that Simon’s model does not in fact produce a simple power law size distribution as the initial element has a dominant first-mover advantage, and will be overrepresented by a factor proportional to the inverse of the innovation probability. The first group’s size discrepancy cannot be explained away as a transient of the model, and may therefore be many orders of magnitude greater than expected. I will demonstrate how Simon’s analysis was correct but incomplete, and expand our alternate analysis to quantify the variability of long term rankings for all groups. I will show some evidence from citation networks that the first-mover advantage is a real feature of certain rich-gets-richer systems. For background, I will provide some historical aspects of Simon and Mandelbrot’s disagreement over the origin of Zipf’s law.
Generalized Kuramoto model in $D$ dimensions: discontinuous transitions and implications for swarms
Contributed talk. Thursday, 10:55 AM - 11:15 AM
Sarthak Chandra, University of Maryland.
Co-authors: Michelle Girvan and Edward Ott, University of Maryland, College Park.

The Kuramoto model is a phase oscillator model that has been used to model the alignment of vectors in 2 dimensions for a wide range of applications. In our work we are primarily interested in using the Kuramoto model to describe the alignment among members of a swarm. We study the Kuramoto model extended to $D$ dimensions. For the Kuramoto model in 2 dimensions with a spread of natural oscillator frequencies, a well known result is that the system is coherent for values of the coupling constant, $K$, greater than a certain critical coupling constant, $K_c > 0$. The strength of this coherence increases continuously as we increase $K$, from an incoherent state for $K \leq K_c$, to coherent states for $K > K_c$. The 3 dimensional Kuramoto model demonstrates a remarkably different behavior in that the magnitude of coherence in the system exhibits a discontinuous jump as $K$ is increased through 0, despite the presence of intrinsic dispersion within the system via a spread of natural rotation frequencies and directions across swarm elements. For any negative value of the coupling constant, the system is incoherent.

Unlike the Kuramoto problem in 2 dimensions, we observe that the behavior of a system of a large number of swarm elements coupled via a Kuramoto-like coupling in 3 dimensions is not low dimensional in the sense of Ott & Antonsen (2008). However, application of our generalized form of the Ott Antonsen ansatz does reduce the dimensionality of the problem when compared with the full system of equations. We use the derived lower dimensional set of equations to analyze the stability of the 3-Dimensional Kuramoto oscillator system and observe that the coherent state demonstrates modes whose stability is weaker than linear.

Thus the 3-Dimensional Kuramoto oscillator system exhibits behavior that is very different when compared with the corresponding 2-Dimensional problem, having implications towards our understanding of alignment of vectors in the often more relevant 3 dimensions, as opposed to the simpler problem in 2 dimensions. We generalize our results beyond the simple Kuramoto model to a wider class of swarm dynamics in high dimensions, and we show that the Ott-Antonsen ansatz can be appropriately generalized for this class of systems. We expect that our results will hence be useful for solving questions involving coupled systems on a sphere in high dimensions, beyond just the simple Kuramoto model.

Solitonic dispersive hydrodynamics
Invited talk. Thursday, 11:15 AM - 11:50 AM
Mark Hoefer, University of Colorado at Boulder.

Long wavelength, hydrodynamic theories abound in physics. Hyperbolic PDEs accurately describe expansion and compression waves until breaking. For many media, the physics at shorter wavelengths are dispersive rather than dissipative, hence higher order dispersive terms are used to describe spatially extended dispersive shock waves (DSWs). Another celebrated solution of equations of this type is the localized solitary wave or soliton but a unified mathematical description of solitons and hydrodynamic flows is lacking. In this talk, new, general soliton-mean field equations are introduced and used to describe the interaction of microscopic solitons with macroscopic hydrodynamics in experiment. This asymptotic theory invokes the scale separation between extended hydrodynamic states and solitons in order to reveal a system of modulation equations for the mean field, the soliton’s amplitude, and a phase. Due to the existence of Riemann invariants, the theory predicts that solitons are trapped by or transmitted through hydrodynamic states. Intriguingly, continuity of the modulation solution implies that the result of solitons incident upon smooth expansion waves or compressive, highly oscillatory DSWs is the same, an effect termed hydrodynamic reciprocity. Quantitative agreement with experiments on interfacial waves between two buoyant, viscous liquids will be shown with broader implications for, e.g., geophysical flows, nonlinear optics, and superfluids.

Temporally periodic neural responses from spatially periodic stimuli
Contributed talk. Thursday, 11:50 AM - 12:10 PM
Jason Pina, University of Pittsburgh.
Co-author: G. Bard Ermentrout, University of Pittsburgh.

Certain static images that have spatial components in a narrow band of wavenumbers have been shown to induce temporally varying neural responses. In one visual illusion, a wheel stimulus with 30-40 spokes caused subjects to perceive part of the static image to be flickering in a regular manner. In pattern sensitive epilepsy, merely reading a book can trigger epileptic seizures for lines of text close to 3 cycles per degree. Similarly, images – including abstract artwork – with peaks in power near 3 cycles per degree are known to cause aversion in
healthy individuals. All of these phenomena have been shown to induce abnormal temporal activity in electroencephalography (EEG) or magnetoencephalography (MEG) recordings. Neural fields have proven useful at modeling the spatiotemporal dynamics of ensembles of neurons and capturing many experimentally observed patterns, such as planar and spiral waves. We are thus motivated to consider a spatially extended neural field model where a static, spatially periodic stimulus is provided as input to the excitatory and inhibitory neural populations. By adjusting system parameters such as the amount of recurrent excitation, we may place the stimulus-free system near a so-called Turing-Hopf bifurcation, where the uniform steady state is spontaneously lost to temporally and spatially periodic patterns with wavenumber \( m \). Simulations and numerical bifurcation analysis for the 1-D system demonstrate the desired resonance, displaying spatially periodic temporal oscillations with very weak stimuli for some wavenumbers while requiring much stronger stimuli for others. By linearizing around the spatially periodic solution at the Turing-Hopf instability, we analytically show that a weak stimulus with wavenumber \( m \) destabilizes the steady state, and find the stability boundary as a function of the recurrent excitation and stimulus strength. Finally, we present a more realistic 2-D system simulated on a GPU that exhibits this strong sensitivity to the spatial frequency of the stimulus. These 2-D simulations also allow us to demonstrate resonance to noisy images with dominant wavenumbers near \( m \), matching experimental findings in visual discomfort.

**Control of unknown chaotic systems with reservoir computing**

Ignite Session A. Thursday, 2:00 PM - 2:05 PM. See also poster A10.

**Daniel M. Canaday**, Ohio State University.

Co-authors: Aaron Griffith and Daniel Gauthier, Ohio State University.

We present a recurrent neural network (RNN) method for controlling an unknown dynamical system. The approach is based on a type of RNN called a reservoir computer (RC) that is distinguished by an untrained, recurrent hidden layer (called the reservoir) and a memoryless output layer that can be rapidly trained by a simple linear regression. We demonstrate that RCs can learn a relationship between an unknown system’s inputs and outputs and that this relationship can be used for control. We demonstrate the utility of this method by stabilizing the well-studied Lorenz 63 system with noise to fixed points and to periodic orbits.

**Stability of spiral waves in cardiac dynamics**

Ignite Session A. Thursday, 2:05 PM - 2:10 PM. See also poster A18.

**Stephanie Dodson**, Brown University.

Co-author: Bjorn Sandstede, Brown University.

Ventricular tachycardia, a dangerous fast-paced heartbeat rate, is a result of a sustained spiral wave rotating on the surface of the heart. After the spiral destabilizes, unorganized electrical activity leads to sudden cardiac arrest the leading natural cause of death in the US. I will present stability results of spiral waves formed in the Karma Model, which is a reaction-diffusion system describing electrical activity in cardiac tissue. My talk will highlight spectral properties of spiral waves formed on bounded disks in reaction-diffusion systems. Absolute and essential spectra of spirals are calculated using asymptotic wave trains, and are compared with point spectra of spirals on large disks. In addition, I will address difficulties that arise in spectral calculations when one or more components of the system are diffusionless.

**A Geometric phase transition in physical networks embedded in 3D**

Ignite Session A. Thursday, 2:10 PM - 2:15 PM. See also poster A16.

**Nima Dehmamy**, Northeastern University.

Co-author: Soodabeh Milanlouei, CCNR Northeastern University.

In many physical networks, from neurons in the brain to 3D integrated circuits or underground hyphal networks, the nodes and links are physical objects unable to cross each other. These non-crossing conditions constrain their layout geometry and affect how these networks form, evolve and function, limitations ignored by the extensive theoretical framework used to characterize real networks. Indeed, the currently used network layout methods are variants of the Force-Directed Layout (FDL) algorithm, which assumes dimensionless nodes and links, hence are inherently unable to reveal the geometry of densely packed physical networks. Here, we develop a modeling framework inspired by manifold dynamics that accounts for the physical reality of nodes and links, allowing us to explore how the non-crossing conditions affect the geometry of the network layout. For small link thicknesses, \( r_L \), we observe a weakly interacting phase where the layout avoids the numerous potential link crossings via local link rearrangements, without altering the overall layout geometry. Once \( r_L \) exceeds a critical threshold, a strongly interacting phase emerges, where multiple geometric quantities, from the total link length to the link curvature, scale with \( r_L \).

We show that the observed transition is driven by ex-
cluded volume interactions, allowing us to analytically derive the transition point. We also find that networks display a solid-like response to stress in the weakly interacting phase, whereas they behave in a gel-like fashion in the strongly interacting phase, illustrating the impact of the observed transition on the physical properties of the network. Overall, we observe a deep universality, finding that the observed scaling properties are independent of the underlying network topology. Finally, we show that the weakly interacting phase offers avenues to 3D print networks, while the strongly interacting phase offers insight on the scaling of densely packed mammalian brains.

**A data-driven approach to computing time-dependent active subspaces in dynamical systems**

Ignite Session A. Thursday, 2:15 PM - 2:20 PM. See also poster A2.

**Izabel P. Aguiar**, University of Colorado at Boulder.
Co-author: Paul G. Constantine, University of Colorado at Boulder.

Active subspaces identify important linear combinations of parameters, enabling more powerful and effective dimension reduction. Although active subspaces give model insight and computational tractability for scalar-valued functions, it is not enough. This analysis does not extend to time-dependent systems. Extending active subspaces to time-dependent systems will enable uncertainty quantification, sensitivity analysis, and parameter estimation for computational models that have explicit dependence on time.

The state-of-the-art method for identifying time-dependent active subspaces is to compute them at individual time steps. Using this approach we identify active subspaces in various engineering and biological dynamical systems. This approach is computationally expensive, however: it requires resampling, computing, and decomposing at every time step. In rapid transients, necessarily small time steps lead to many more computations.

To reduce computational cost we implement Dynamic Mode Decomposition (DMD) [Kutz et al., 2016] and Sparse Identification for Nonlinear Dynamical Systems (SINDy) [Brunton et al., 2016] to reconstruct and predict future active subspaces. We also derive analytical forms of time-dependent active subspaces for time-dependent outputs of two linear parameterized dynamical systems. This analysis and computation inform visualization and insight of parameter dependence in various dynamical systems.

**Predicting full and partial synchronization of Kuramoto oscillators on arbitrary networks**

Ignite Session A. Thursday, 2:20 PM - 2:25 PM. See also poster A20.

**Brady M. Gilg**, Arizona State University.
Co-author: Dieter Armbruster, Arizona State University.

The network Kuramoto model is a simple model of phase oscillators embedded in a network. Each oscillator is assumed to have nonidentical internal dynamics, as well as external forcing mediated by a coupling function from each oscillator to its neighbors. Two fascinating emergent phenomena exhibited by the model are those of "full synchronization" and "partial synchronization", occurring at higher and lower strengths of coupling. In the former, all oscillators maintain an identical constant frequency. In the latter, the Kuramoto network segments into clusters of oscillators which are frequency synchronized within themselves, but not to external clusters. The current literature covers a number of results which establish upper bounds on the critical coupling required for full synchronization, but very few lower bounds, and nearly no results pertaining to partial synchronization. Here, we introduce a Kuramoto network reduction technique and show how this reduction leads to a weighted graph partitioning problem. The partitioning problem can then be solved approximately with known optimization techniques utilizing spectral graph theory. Finally, we provide numerical evidence that this technique produces a lower bound on the critical coupling and predicts the partially synchronized behavior of a Kuramoto system with high accuracy.

**In silico analysis of antibiotic-induced C. difficile infection**

Ignite Session A. Thursday, 2:25 PM - 2:30 PM. See also poster A28.

**Eric Jones**, UC Santa Barbara.
Co-author: Jean Carlson, UC Santa Barbara.

We study antibiotic-induced C. difficile infection (CDI), caused by the toxin-producing C. difficile (CD), and implement clinically-inspired simulated treatments in a computational framework that synthesizes a generalized Lotka-Volterra (gLV) model with SIR modeling techniques. The gLV model uses parameters derived from an experimental mouse model, in which the mice are administered antibiotics and subsequently dosed with CD. We numerically identify which of the experimentally measured initial conditions are vulnerable to CD colonization, then formalize the notion of CD susceptibility analytically. We simulate fecal transplanta-
tion, a clinically successful treatment for CDI, and discover that both the transplant timing and transplant donor are relevant to the efficacy of the treatment, a result which has clinical implications. We incorporate two nongeneric yet dangerous attributes of CD into the gLV model, sporulation and antibiotic-resistant mutation, and for each identify relevant SIR techniques that describe the desired attribute. Finally, we rely on the results of our framework to analyze an experimental study of fecal transplants in mice, and are able to explain observed experimental results, validate our simulated results, and suggest model-motivated experiments.

**Galaxy-Like organization of floaters on Faraday waves**

In experiments of standing waves in a channel, we observe a complex pattern of tiny floating particles. Starting with an homogeneous distribution of floating particles, we drive the container with a vertical oscillation. After standing waves develop (because of Faraday instability), particles start to agglomerate in elliptical patterns with arms on the sides. The arms structure resembles the one encountered in some Galaxies.

Floater patterns appear in well defined positions in the channel, around the anti-nodes of the waves. Patterns rotate with a well defined velocity that scales with wave amplitude. The arms also grow with the wave amplitude, despite they appear at a threshold different from the one of the circulation. We interpret our observations as follows: viscosity drives a steadily circulating streaming flow [with the same origin as in Perinet et al. JFM, 2017]. The circulation couples with the periodic flow induced by the standing wave, generating the folding of the material lines into arms. One arm appears after each wave period.

Although standing waves are not supposed to transport matter, it was reported that floating particles cumulate on the nodes or anti-nodes of standing waves, depending on the wetting properties and inertia of particles [Falkovich et al. Nature, 2005]. Latter, in similar experiments it was observed that the covering fraction of the surface is also relevant to trigger more complex collective scenarios [Sanli et al. PRE 2014]. Aside from these mechanisms related to the physical properties of particles, our observations suggest a mechanism more related with the geometry of the flow itself.

**Tracking hidden nodes in networks using adaptive filtering**

In network analysis, we are often confronted with the problem of hidden nodes, or parts of a network that we have no measurements, and thus no knowledge of. These hidden nodes act as unknown drivers and can have a significant influence on the network state. Detecting their presence, or more specifically the parts of the network they are influencing, is thus critical. We propose an adaptive filtering-based method for the detection and tracking of hidden nodes in complex networks. By viewing the hidden nodes as a problem of missing variables in the model fitting process, we show that the filter estimated system noise covariance allows us to localize the effects of the hidden node and track changes in its driving strength over time.

**Making an effort to listen: mechanical amplification by ion channels and myosin molecules in hair cells of the inner ear**

As the gateway to human communication, the sense of hearing is of enormous importance in our lives. Hearing commences with the capture of sound energy by hair cells, the ear’s sensory receptors, which convert that energy into electrical signals that the brain can then interpret. Each hair cell is a cylindrical epithelial cell surrounded by a hair bundle, an erect cluster of 20,000 rigid, actin-filled rods termed stereocilia. Whether a sensory organ is specialized for the detection of sound, acceleration, or water movement, a mechanical force deflects the hair bundle and thereby excites the hair cell and its associated nerve fibers.

Uniquely among our sensory receptors, the hair cell is not a passive recipient of stimuli, but instead uses an active process to enhance its inputs. This active process amplifies mechanical stimuli by as much as a thousandfold, thus greatly increasing our sensitivity to weak sounds. When this process fails, we become hard of hearing. Amplification is accompanied by frequency tuning, which restricts each hair cell’s response to a narrow frequency band. This feature endows our hearing with a frequency resolution of less than 0.2%, one thirtieth of the interval between successive piano keys. If the active process deteriorates, we grow less sensitive to subtle differences in frequency and there-
fore suffer a diminished ability to discriminate sound sources. The active process produces a compressive nonlinearity that renders the ear sensitive to sounds over a millionfold range of amplitude or an astonishing trillionfold range in power. By enhancing weak stimuli and suppressing strong ones, this feature allows us to enjoy an instrumental soloist as comfortably as a full orchestra hundreds of times as loud. Finally, the active process can be so exuberant as to become unstable; as a result, in a very quiet environment most normal ears spontaneously emit sound! These otoacoustic emissions are idiosyncratic to individual ears, but in the absence of auditory damage remain stable over time.

As a result of the cooperative gating of mechanically sensitive ion channels, a hair bundle can be dynamically unstable: the relation between the bundle’s displacement and the force required to accomplish it possesses two stable fixed points separated by a region of negative stiffness. Experiments on individual hair bundles indicate that the bundle’s operation near this instability - a Hopf bifurcation - accounts for the four characteristics of the active process. As with the region of negative resistance in a tunnel diode, this situation can foster amplification or oscillation. Whereas the diode is biased with a voltage, the hair bundle is pushed into its region of instability by molecular motors, specifically the myosin molecules associated with adaptation of the transduction apparatus to sustained stimuli.

The mathematics and mechanics of brain morphogenesis
Invited talk. Thursday, 4:25 PM - 5:00 PM
Alain Goriely, Oxford University.

The human brain is an organ of extreme complexity, the object of ultimate intellectual egocentrism, and a source of endless scientific challenges. Its intricate folded shape has fascinated generations of scientists and has, so far, defied a complete description. How does it emerge? How is its shape related to its function? How does the skull grow to accommodate the brain? In this talk, I will review our current understanding of brain morphogenesis and its unique place within a general mathematical theory of biological growth. In particular, I will present simple models for basic pattern formation and show how they help us understand axonal growth, brain folding, and skull formation.

Drift in terrestrial orbits
Contributed talk. Thursday, 5:00 PM - 5:20 PM
Jerome Daquin, RMIT, Australia.
Co-authors: Aaron J. Rosengren, University of Arizona; Ioannis Gkolias, Politecnico di Milano, Italy.

Recent endeavors have highlighted the coexistence of stable and chaotic motions in the secular (i.e., long-term) motion of artificial satellites of the Earth perturbed by the effect of lunar and solar gravity. This concerns a wide range of the perturbing parameter, a function of the mean orbital distance from the central body. The dynamical understanding of the long-term properties of these orbits helps to address the issues raised by the mitigation of the perennial motion of space debris.

In this contribution, we depart from explanations supporting the mechanisms of chaos to focus rather on its consequences in terms of transport. Thanks to a computationally efficient vectorial model based on the Milankovitch elements, we are able to calculate high-resolution Fast Lyapunov Indicator (FLI) maps (one order of magnitude more resolved than existing maps in the literature) to delimit the fine topology of the phase space and discriminate regularities. This allows us to extract and select interesting initial conditions close to hyperbolic boarders, for which we proceed to analyze and establish the existence of drifts. We then present results concerning the computation of diffusion-like coefficients based on ensemble averages.
Nonlinear waves in granular crystals: mathematical analysis, numerical computations and physical experiments
Invited talk. Friday, 8:45 AM - 9:20 AM
Panos Kevrekidis, University of Massachusetts.

In this talk, we will provide an overview of results in the setting of granular crystals, consisting of beads interacting through Hertzian contacts. We will start from the simplest one dimensional settings, where we will show that there exist three prototypical types of coherent nonlinear waveforms: shock waves, traveling solitary waves and discrete breathers. The latter are time-periodic, spatially localized structures. For each one, we will analyze the existence theory, presenting connections to prototypical models of nonlinear wave theory, such as the Burgers equation, the Korteweg-de Vries equation and the nonlinear Schrodinger (NLS) equation, respectively. We will also explore the stability of such structures, presenting some explicit stability criteria analogous to the famous Vakhitov-Kolokolov criterion in the NLS model. Finally, for each one of these structures, we will complement the mathematical theory and numerical computations with state-of-the-art experiments, allowing their quantitative identification and visualization. Finally, ongoing extensions of these themes will be briefly touched upon, most notably in higher dimensions, in heterogeneous or disordered chains and in the presence of damping and driving; associated open questions will also be outlined.

Topological analysis of experimental recordings of ventricular fibrillation
Contributed talk. Friday, 9:20 AM - 9:40 AM
Daniel Gurevich, Georgia Tech.
Co-authors: Conner Herndon, Flavio H. Fenton, and Roman O. Grigoriev, Georgia Tech.

Spiral wave breakup caused by dispersion of tissue refractoriness has long been believed to play a key role in maintaining complex cardiac rhythms such as atrial and ventricular fibrillation. To test this hypothesis, we have developed a level-set based method that can accurately and reliably extract the temporally and spatially resolved positions of wavefronts, wavebacks, and phase singularities from noisy optical mapping data. The utility of this method was illustrated by analyzing optical mapping voltage data during ventricular fibrillation in a Langendorff-perfused pig heart. A recent topological analysis has shown that, in two dimensions, there are four distinct mechanisms which increase the complexity of the excitation pattern (wave breakup being one of them) and four distinct mechanisms which decrease its complexity. This analysis predicted that wave coalescence plays a more important role in maintaining fibrillation than wave breakup. A similar topological analysis of our experimental recordings provides supporting evidence for this theoretical prediction.

Synchronization by uncorrelated noise: interacting collective oscillations in networks of oscillator networks
Contributed talk. Friday, 10:55 AM - 11:15 AM
Hermann Riecke, Northwestern University.
Co-authors: John H. Meng, Northwestern University.

Oscillators coupled in a network often synchronize with each other, resulting in a collective oscillation or coherent population rhythm. How do multiple such rhythms interact with each other? We show that for strong, inhibitory coupling certain types of rhythms become synchronized by noise. Importantly, in contrast to the case of stochastic synchronization, noise synchronizes the rhythms even if the noisy inputs to different oscillators are completely uncorrelated. Key for the synchrony across networks is the reduced synchrony within the networks: it substantially increases the frequency
range across which the networks can be entrained by other networks or by periodic pacemaker-like inputs. More specifically, the noise-enhanced synchronizability of these rhythms arises from a network mechanism: it requires a minimal network size and emerges from the variability in the number of oscillators that participate in the collective oscillation and the resulting variability of the oscillation frequency. We condense this new synchronization mechanism into a simple iterated map, which captures the reverse period-doubling bifurcation that leads to the synchronization. The synchronization mechanism is very robust. We demonstrate it for networks comprised of different classes of neuron models with different synaptic couplings and for different network connectivities. Moreover, we show that the same mechanism can synchronize collective oscillations in networks of relaxation oscillators that interact through a fast-diffusion mechanism that is related to quorum sensing. The synchronization of different population rhythms may be particularly relevant for brain rhythms, for which we find that noise can enhance the learning by spike-timing-dependent plasticity.

**Host-pathogen dynamics during malaria infection**

Invited talk. Friday, 11:15 AM - 11:50 AM

**Steven Haase**, Duke University.

The hallmark of malaria infection is a cyclic fever with a period of 48 hours, and in rarer cases fever cycles can be 24 or 72 hours, depending on the species. Because each of these periods is a multiple of a circadian period (24 hours), it has been suspected that circadian clocks play a prominent role in the infection. However, little is known about how the host circadian clock may influence the dynamics of infection and periodic fevers. We have identified molecular signatures indicating that the malaria parasite has an intrinsic clock mechanism that controls an oscillating program of phase-specific expression. We have also collected time-series data from infected patients that suggest the parasite and host circadian clock mechanisms are coupled during the intraerythrocyte cycle of the infection. We propose that malaria parasites have evolved strategies to synchronize their intraerythrocyte cycle with the host circadian clock to circumvent host defenses that are under circadian control.

**Optimal bounds and extremal trajectories for time averages in nonlinear dynamical systems**

Contributed talk. Friday, 11:50 AM - 12:10 PM

**Charles Doering**, University of Michigan.

Co-authors: Ian Tobasco, University of Michigan; David Goluskin, University of Victoria, Canada.

For any quantity of interest in a system governed by nonlinear differential equations, it is natural to seek the largest (or smallest) long-time average among solution trajectories. Upper bounds can be proved *a priori* using auxiliary functions, the optimal choice of which is a convex optimization. We show that the problems of finding maximal trajectories and minimal auxiliary functions are strongly dual. Thus, auxiliary functions provide arbitrarily sharp upper bounds on maximal time averages. They also provide volumes in phase space where maximal trajectories must lie. For polynomial equations, auxiliary functions can be constructed by semidefinite programming which we illustrate using the Lorenz and Kuramoto-Sivashinsky equations.

**Control of macroscopic neuronal network activity is optimal at criticality**

Ignite Session B. Friday, 2:00 PM - 2:05 PM. See also poster B42.

**Kathleen Finlinson**, University of Colorado at Boulder.

Co-authors: Juan G. Restrepo, Daniel B. Larremore, University of Colorado at Boulder.

There is growing evidence that the cortex operates at a critical state where the strength of excitatory and inhibitory neurons is precisely balanced. Recent research has shown that many properties related to information processing are optimized at criticality, including dynamic range, information transmission, and the variability of synchronization. By analyzing neuronal network models, we show here that, in addition, the macroscopic activity of the network is most easily controlled at criticality.

We first analyze a simple binary neuron model, where nodes may be either active or quiescent at each time. The balance in the relative numbers and synapse strengths of excitatory and inhibitory nodes determines whether excitations are amplified, maintained, or decay on average (corresponding to supercritical, critical, and subcritical states, respectively). We consider the problem of controlling the total network activity to a given target value, using a feedback loop with either global or local information. We show numerically and theoretically that the control error, averaged across a range of targets, is minimized when the network is critical. We validate our findings for the binary neuron model.
dependent frictional strength has a direct and subordinate role in controlling the state (or history) of a system. Such a rate-and state-dependent frictional strength depends on instantaneous slip rate and its state (or history). Such a rate-and state-dependent frictional strength has a direct and sub-
sequent evolutionary response towards (logarithmic) changes in slip velocity, magnitudes of which are controlled by parameters $a$ and $b$ respectively [e.g., Ruina 1983]. Those frictional properties dictate whether a slip diffuses on the fault (if $a > b$) or diverge within a compact support (if $a < b$, our focus here). Latter is ensured if the wavelength of perturbations to steady-state sliding exceeds a critical lengthscale.

Governing evolution equations, being invariant under time translation and amplitude scaling, suggests that that the blow-up might occur as $v(x,t) = W(x,t)/t_f$, where $t_f = (t_o - t)$ is the time to instability. The diverging slip rate approaches a self similar profile, if $t_f \partial W/\partial t$ attains a steady state and hence can be obtained using $t_f \partial W/\partial t = 0$ a priori.

Further, setting $t_f \partial W/\partial t = 0$ a priori could lead to multiple self similar solutions in situations where friction parameters are non-uniformly distributed [Ray and Viesca, 2017]. However, only subset of them are attainable for a which perturbation grows at a rate $\sim o(t_f^{-1})$ as $t_f \to 0$. Further, the length scales over which frictional properties vary over the fault could alter the attainability of the self similar solutions. We develop a low parameter model with $\alpha$ and $\kappa$ respectively controlling the magnitude and length scale of heterogeneity. Similarity solutions at specific regions on fault gain or lose stability (as pitchfork and transcritical bifurcations) as $\alpha$ and/or $\kappa$ is varied. We conclude that only specific regions on faults an unstable slip could nucleate. Such preferred earthquake nucleation locations are otherwise lost when frictional heterogeneities are avoided for model simplification purposes.

**Chaotic advection in active nematics**
Ignite Session B. Friday, 2:05 PM - 2:10 PM. See also poster B23.

**Eric Roberts**, UC Merced.
Co-authors: Kevin Mitchell, Amanda Tan, and Linda Hirst, UC Merced.

The recent surge of research into active materials is an exciting development in soft matter physics. Unlike traditionally studied fluids, active fluids are not in equilibrium. Instead, they continuously consume energy to generate internal motion, which can subsequently produce large-scale flows and rich emergent dynamical structures, such as moving topological defects. These moving defects can wind around one another to generate chaotic mixing. We report here on experimental and theoretical work on a biologically inspired active nematic liquid crystal. Densely packed microtubules slide antiparallel to each other at a controlled rate due to kinesin molecular motors. The resulting chaotic advection is studied experimentally using the tools of particle tracking, particle image velocimetry, and fluorescence imaging of labeled tracers. Experimental data are analyzed and interpreted in the context of topological dynamics, thereby bridging the fields of chaotic advection and active fluids. We focus on the topological entropy, measured from the braiding of tracer trajectories and on the local Lyapunov exponent, measured from the divergence of neighboring tracers.

**Self-similar solutions of diverging slip rate on faults with heterogeneous friction**
Ignite Session B. Friday, 2:10 PM - 2:15 PM. See also poster B21.

**Sohom Ray**, Tufts University.
Co-author: Robert C. Viesca, Tufts University.

Frictional sliding of an extended system coupled with its deformation is ubiquitous. One of them is a geological fault, an interface within a continuum across which relative displacement (slip along the interface) may occur. A slip distribution instantaneously leads to quasistatic changes in shear traction on the fault, which adds to the traction due to any external forcing. Slip occurs when and where fault’s shear stress equals its frictional strength.

Rock friction experiments suggest that, contrary to the popular belief of constant friction, a fault’s frictional strength depends on instantaneous slip rate and its state (or history). Such a rate-and state-dependent frictional strength has a direct and sub-
sequent evolutionary response towards (logarithmic) changes in slip velocity, magnitudes of which are controlled by parameters $a$ and $b$ respectively [e.g., Ruina 1983]. Those frictional properties dictate whether a slip diffuses on the fault (if $a > b$) or diverge within a compact support (if $a < b$, our focus here). Latter is ensured if the wavelength of perturbations to steady-state sliding exceeds a critical lengthscale.

Governing evolution equations, being invariant under time translation and amplitude scaling, suggests that that the blow-up might occur as $v(x,t) = W(x,t)/t_f$, where $t_f = (t_o - t)$ is the time to instability. The diverging slip rate approaches a self similar profile, if $t_f \partial W/\partial t$ attains a steady state and hence can be obtained using $t_f \partial W/\partial t = 0$ a priori.

Further, setting $t_f \partial W/\partial t = 0$ a priori could lead to multiple self similar solutions in situations where friction parameters are non-uniformly distributed [Ray and Viesca, 2017]. However, only subset of them are attainable for a which perturbation grows at a rate $\sim o(t_f^{-1})$ as $t_f \to 0$. Further, the length scales over which frictional properties vary over the fault could alter the attainability of the self similar solutions. We develop a low parameter model with $\alpha$ and $\kappa$ respectively controlling the magnitude and length scale of heterogeneity. Similarity solutions at specific regions on fault gain or lose stability (as pitchfork and transcritical bifurcations) as $\alpha$ and/or $\kappa$ is varied. We conclude that only specific regions on faults an unstable slip could nucleate. Such preferred earthquake nucleation locations are otherwise lost when frictional heterogeneities are avoided for model simplification purposes.

**Non-universal system dynamics near excitatory-inhibitory balance in biophysical neuronal network**
Ignite Session B. Friday, 2:15 PM - 2:20 PM. See also poster B36.

**Jiaying Wu**, University of Michigan, Ann Arbor.
Co-authors: Sara Aton, Victoria Booth, and Michal Zochowski, University of Michigan, Ann Arbor.

It has been postulated that the brain networks universally organize themselves around dynamical state characterized by balanced excitation and inhibition, where the relative contributions of excitatory and inhibitory activities are generally equal. It has been shown that there are dynamical advantages in being in that state and it was hypothesized that the normal cognitive function is mediated by network wide activity near that state. At the same time, it is widely assumed that abnormal E/I balance is implicated to underlie generation of brain pathologies (e.g. epilepsy). However, the mechanisms of formation nor universal implications of
E/I balance have never been studied in detail. Furthermore, to complicate things, it is nearly impossible to measure it experimentally, a number of ad-hoc metrics is used that are to provide substitute for that measurement. These take into account relative neuronal frequency, firing pattern, synaptic current or local field potential spectrum etc. Here we report detailed investigation on the dynamics of the biophysical neuronal network near states that are characterized by balanced excitatory and inhibitory synaptic currents. Our results show that ‘E-I balance’ does not correspond to single, universal network dynamical state, but multiple dynamical states with significantly different overall functional properties. The properties of these states depend on relative synaptic strength, network topology and other parameters. Furthermore, counterintuitively, when increasing the excitatory synaptic strength, the E-I ratio does not monotonically increase, and only for strongly coupled system, the network is universally bounded near to the balanced state. In conclusion, our results show that E-I balance is not a unique network state and can correspond to various dynamics in the network, which may result in critically different network function or pathology.

**Persistent homology and spatial population dynamics**
Ignite Session B. Friday, 2:20 PM - 2:25 PM. See also poster B31.
Laura Storch, College of William and Mary.
Co-author: Sarah Day, College of William and Mary.

Chaotic dynamics have been directly detected in myriad and diverse biological systems, such as fisheries, infectious disease spread, and tree growth dynamics. Because chaotic systems can be extremely sensitive to changes in the system, it becomes particularly important to better understand chaotic populations, especially in the context of our rapidly changing global climate. Here, we analyze the dynamics of a coupled-patch, spatially distributed density-dependent population with growth and dispersal phases. The population model is discrete in both space and time, and exhibits complex spatial and temporal dynamics. We employ persistent homology to analyze spatial patterns in the population over time. These topological measurements can be used to quantify spatial structures in the model, and preliminary results suggest that they can identify changes in spatial structure preceding global extinction events.

**Identical synchronization of nonidentical oscillators: when only birds of different feathers flock together**
Ignite Session B. Friday, 2:25 PM - 2:30 PM. See also poster B41.
Yuanzhao Zhang, Northwestern University.
Co-author: Adilson E. Motter, Northwestern University.

An outstanding problem in the study of networks of heterogeneous dynamical units concerns the development of rigorous methods to probe the stability of synchronous states when the differences between the units are not small. Here, we address this problem by presenting a generalization of the master stability formalism that can be applied to heterogeneous oscillators with large mismatches. Our approach is based on the simultaneous block diagonalization of the matrix terms in the variational equation, and it leads to dimension reduction that simplifies the original equation significantly. This new formalism allows the systematic investigation of scenarios in which the oscillators need to be nonidentical in order to reach an identical state, where all oscillators are completely synchronized. In the case of networks of identically coupled oscillators, this corresponds to breaking the symmetry of the system as a means to preserve the symmetry of the dynamical state - a recently discovered effect termed asymmetry-induced synchronization (AISync). Our framework enables us to identify communication delay as a new and potentially common mechanism giving rise to AISync, which we demonstrate using networks of delay-coupled Stuart-Landau oscillators. The results also have potential implications for control, as they reveal oscillator heterogeneity as an attribute that may be manipulated to enhance the stability of synchronous states.

**Fast cheater migration stabilizes coexistence in a public goods dilemma on networks**
Ignite Session B. Friday, 2:30 PM - 2:35 PM. See also poster B39.
Glenn S. Young, Pennsylvania State University.
Co-author: Andrew Belmonte, Pennsylvania State University.

Cooperation is frequently considered an unsustainable strategy: if an entire population is cooperating, each individual can increase its overall fitness by choosing not to cooperate, thereby still receiving all the benefit of its cooperating neighbors while no longer expending its own energy. Observable cooperation in naturally-occurring public goods games is consequently of great interest, as such systems offer insight into both the emergence and sustainability of cooperation. Here we consider a population that obeys a public goods game
on a network of discrete regions (that we call nests), between any two of which individuals are free to migrate. We construct a system of piecewise-smooth ordinary differential equations that couple the within-nest population dynamics and the between-nest migratory dynamics. Through a combination of analytical and numerical methods, we show that if the workers within the population migrate sufficiently fast relative to the cheaters, the network loses stability first through a Hopf bifurcation, then a torus bifurcation, after which one or more nests collapse. Our results indicate that fast moving cheaters can act to stabilize worker-cheater coexistence within network that would otherwise collapse.

Slow manifolds in the aerodynamic descent of animals and plants
Ignite Session B. Friday, 2:35 PM - 2:40 PM. See also poster B12.
Gary K. Nave, Virginia Tech.
Co-author: Shane D. Ross, Virginia Tech.

A large variety of animals exhibit the capacity for gliding flight, including but not limited to species of squirrels, lizards, fish, ants, and snakes. Historically, to understand these animals, researchers have considered equilibrium glide angle, defined by the mean lift-to-drag ratio. However, in a glide, most of the animal’s motion occurs away from equilibrium. To more accurately understand the glide, we consider a 2 degree-of-freedom model, which considers lift and drag coefficients as functions of angle of attack and the animal’s pitch angle with respect to the ground as a control parameter. In this model, we find that most of the dynamics occur along a slow manifold in the presence of a fast, superstable direction. Because all trajectories collapse onto this slow manifold, we may consider it as a higher-dimensional analog for terminal velocity, which is itself a codimension-one structure in a 1 degree-of-freedom model. In this work, we present a variety of methods for calculating this terminal velocity manifold, which give new insight to its identity. We also prescribe the pitch dynamics in extended phase space and re-consider gliding and fluttering dynamics as occurring on a two-dimensional terminal velocity manifold embedded in a three-dimensional system. These phase space structures may be leveraged for efficient control strategies of engineered aerial dispersal systems in the future.

Data-driven discovery of governing equations and physical laws
Invited Talk, Friday, 2:50 PM - 3:25 PM
Nathan Kutz, University of Washington.

The emergence of data methods for the sciences in the last decade has been enabled by the plummeting costs of sensors, computational power, and data storage. Such vast quantities of data afford us new opportunities for data-driven discovery, which has been referred to as the 4th paradigm of scientific discovery. We demonstrate that we can use emerging, large-scale time-series data from modern sensors to directly construct, in an adaptive manner, governing equations, even nonlinear dynamics and PDEs, that best model the system measured using modern regression and machine learning techniques. We can also discover nonlinear embeddings of the dynamics using Koopman theory and deep neural network architectures. Recent innovations also allow for handling multi-scale physics phenomenon and control protocols in an adaptive and robust way. The overall architecture is equation-free in that the dynamics and control protocols are discovered directly from data acquired from sensors. The theory developed is demonstrated on a number of canonical example problems from physics, biology and engineering.

Glassy dynamics of inference
Invited talk, Friday, 4:05 PM - 4:40 PM
Cris Moore, Santa Fe Institute.

Belief propagation is a popular algorithm for statistical inference, which dates back to the Turing Award-winning work of Judea Pearl. But like any iterative algorithm, it can have multiple fixed points and basins of attraction. I’ll describe a phase transition in one of my favorite inference problems - finding communities in social or biological networks - where inference becomes exponentially hard because the accurate fixed point has an exponentially small basin of attraction. This is analogous to phenomena in spin glasses, where the crystalline ground state becomes dynamically inaccessible, hidden behind a free energy barrier.

The sausage-string structure of mode-locking regions of piecewise-linear maps
Contributed talk, Friday, 4:40 PM - 5:00 PM
David Simpson, Massey University, New Zealand.

Mode-locking regions are subsets of parameter space where a dynamical system is entrained to a fixed frequency or rotation number. In two-parameter bifurcation diagrams they appear as narrow regions ordered by
rotation number. For piecewise-linear maps they have pinch points, called shrinking points, and an overall structure that resembles a string of sausages. This has been identified in models of diverse systems, including power converters, neurons, and economics, and remains incompletely understood. In this talk I will explain how each shrinking point organizes the bifurcation structure locally. A handful of key scalar quantities assigned to a shrinking point govern the relative size, properties, and arrangement of nearby mode-locking regions. In sectors radiating from a shrinking point, periodic, quasiperiodic, and chaotic dynamics are accurately captured by a one-dimensional skew sawtooth map.

Using optimal stretching to forecast advection-reaction-diffusion dynamics
Contributed talk, Friday, 5:00 PM - 5:20 PM
Douglas H. Kelley, University of Rochester.
Co-authors: Thomas D. Nevins, Jeff Tithof, Jinge Wang, and Rony O. Colón, University of Rochester.

Complex dynamics emerge in chemical and biological systems where the effects of advection (flow), reaction (growth), and diffusion (molecular spreading) are combined. Tools to forecast advection-reaction-diffusion systems would allow prediction of phytoplankton growth in Earth’s oceans, construction and deconstruction of cellular scaffolding in crawling cells, and — if feedback could be included — combustion. I will talk about using optimal stretching as such a tool. In recent experiments, my team and I have found that the regions of an ARD system most likely to be reacted are the ones where the Lagrangian stretching is neither too weak nor too strong, but falls in an optimal range. A local, physical mechanism explains the effect, which occurs in both open and closed flows, though in other ways their topology and dynamics are distinct. Locating regions of optimal stretching gives a simple algorithm for forecasting growth in advection-reaction-diffusion systems. I will show examples and close with a few ideas for future work.

Saturday

The mathematics of taffy pulling
Invited talk, Saturday, 8:45 AM - 9:20 AM
Jean Luc Thiffeault, University of Wisconsin.

Taffy is a type of candy made by repeated ‘pulling’ (stretching and folding) a mass of heated sugar. The purpose of pulling is to get air bubbles into the taffy, which gives it a nicer texture. Until the late 19th century, taffy was pulled by hand, an arduous task. The early 20th century saw an avalanche of new devices to mechanize the process. These devices have fascinating connections to the topological dynamics of surfaces, in particular with pseudo-Anosov maps. Special algebraic integers such as the Golden ratio and the lesser-known Silver ratio make an appearance, as well as more exotic numbers. We examine different designs from a mathematical perspective, and discuss their efficiency.

Collective mechanical adaptation of honeybee swarms
Contributed talk, Saturday, 9:20 AM - 9:40 AM
Orit Peleg, University of Colorado at Boulder.
Co-authors: Jacob M. Peters, Mary K. Salecedo, Lakshminarayanan Mahadevan, Harvard University.

Honeybee (Apis mellifera) swarms form clusters made solely of bees attached to each other, forming pendant structures on tree branches. These clusters can be hundreds of times the size of a single organism. How these structures are stably maintained under the influence of static gravity and dynamic stimuli (e.g. wind) is unknown. To address this, we created pendant conical clusters attached to a board that was shaken with varying amplitude, frequency and total duration. Our observations show that horizontally shaken clusters spread out to form wider, flatter cones, i.e. the cluster adapts to the dynamic loading conditions, but in a reversible manner - when the loading is removed, the cluster recovers its original shape, slowly. Measuring the response of a cluster to a sharp pendular excitation before and after it adapted shows that the flattened cones deform less and relax faster than the elongated ones, i.e. they are more stable mechanically. We use particle-based simulations of a passive assemblage to suggest a behavioral hypothesis that individual bees respond to local variations in strain. This behavioral response improves the collective stability of the cluster as a whole at the expense of increasing the average mechanical burden experienced by the individual. Simulations using this rule explain our observations of adaptation to horizontal shaking. The simulations also suggest that vertical shaking will not lead to significant differential strains and thus no adaptation. To test this, we shake the cluster vertically and find that indeed there is no response to this stimulus. Altogether, our results show how an active, functional super-organism structure can respond adaptively to dynamic mechanical loading by changing its morphology to achieve better load sharing.
The role of tissue biophysics in cancer
Invited talk, Saturday, 9:40 AM - 10:15 AM
Kandice Tanner, NIH.

Transformation of the physical microenvironment including changes in mechanical stiffness of the extracellular matrix (ECM) may be one of the crucial factors that drives cancer progression. In addition to tissue mechanics, the surface topography of the ECM microenvironment has been shown to modulate gene expression. Simply put, how do changes in the physical microenvironment drive cancer progression? 3D culture models can approximate in vivo architecture and signaling cues, allowing for real time characterization of cell-ECM dynamics. We developed tissue mimetics that recreate the complex in vivo geometries while independently controlling bulk stiffness and ECM ligand density. We also developed tools that allow us to resolve and quantitate minute forces that cells sense in the local environment (on the order of microns) within thick tissue (in mm). Using these methods, we are able to dissect the contributions of the physical properties from those due to chemical properties on cell fate as it relates to malignancy and normal tissue homeostasis. Finally, we validated our in vitro findings in an in vivo model using zebrafish as our model for metastasis.

Structural and functional redundancy in biological networks
Contributed talk, Saturday, 10:55 AM - 11:15 AM
Alice C. U. Schwarze, University of Oxford, UK.
Co-authors: Mason A. Porter, UCLA; Jonny Wray, e-Therapeutics plc, UK.

Several scholars of evolutionary biology have suggested that functional redundancy (also known as biological “degeneracy”) is important for robustness of biological networks. Structural redundancy indicates the existence of structurally similar subsystems that can perform the same function. Functional redundancy indicates the existence of structurally different subsystems that can perform the same function. For networks with Ornstein–Uhlenbeck dynamics, Tononi et al. [Proc. Natl. Acad. Sci. U.S.A. 96, 3257–3262 (1999)] proposed measures of structural and functional redundancy that are based on mutual information between subnetworks. For a network of $n$ vertices, an exact computation of these quantities requires $O(n!)$ time. We derive expansions for these measures that one can compute in $O(n^3)$ time. We use the expansions to compare the contributions of different types of motifs to a network’s structural and functional redundancy. We compute structural and function redundancy for protein-interaction networks and find that these networks have larger functional redundancy than corresponding realisations of several random-graph models.

Using computational fluid dynamics to understand the neuromechanics of jellyfish swimming
Invited talk, Saturday, 11:15 AM - 11:50 AM
Laura Miller, University of North Carolina.

Recent advancements in computational fluid dynamics have enabled researchers to efficiently explore problems that involve moving elastic boundaries immersed in fluids for problems such as cardiac fluid dynamics, fish swimming, and the movement of bacteria. These advances have also made modeling the interaction between a fluid and a neuromechanical model of an elastic organism feasible. This project focuses on the development and implementation of such models for the pulsation and movement of jellyfish bells. We leverage existing computational algorithms for fluid-structure interactions and couple this technology to living boundaries. The model integrates feedback between the conduction of action potentials, the contraction of muscles, the movement of tissues, and fluid motion.

Intermittent many-body dynamics at equilibrium
Contributed talk, Saturday, 11:50 AM - 12:10 PM
David K. Campbell, Boston University.
Co-authors: Carlo Danieli and Sergej Flach, Center for Theoretical Physics of Complex Systems, Institute for Basic Science, Korea.

The equilibrium value of an observable defines a manifold in the phase space of an ergodic and equipartitioned many-body system. A typical trajectory pierces that manifold infinitely often as time goes to infinity. We use these piercings to measure both the relaxation time of the lowest frequency eigenmode of the Fermi-Pasta-Ulam chain (FPU), as well as the fluctuations of the subsequent dynamics in equilibrium. The dynamics in equilibrium is characterized by a power-law distribution of excursion times far off equilibrium, with diverging variance. Long excursions arise from sticky dynamics close to q-breathers localized in normal mode space. Measuring the exponent allows to predict the transition into nonergodic dynamics. We generalize our method to Klein-Gordon lattices (KG) where the sticky dynamics is due to discrete breathers localized in real space.
Snaking in dimensions 1+Epsilon
Contributed talk, Saturday, 2:00 PM - 2:20 PM
Jason Bramburger, Brown University.
Co-author: Bjorn Sandstede, Brown University.

The Swift-Hohenberg equation is a widely studied partial differential equation which is known to support a variety of spatially localized structures. The one-dimensional equation exhibits spatially localized steady-state solutions which give way to a bifurcation structure known as snaking. That is, these solutions bounce between two different values of the bifurcation parameter while ascending in norm. The mechanism that drives snaking in one spatial dimension is now well-understood, but recent numerical investigations indicate that upon moving to two spatial dimensions the related radially-symmetric spatially-localized solutions take on a significantly different snaking structure which consists of three major components. To understand this transition we apply a dimensional perturbation in an effort to use well-developed methods of perturbation theory and dynamical systems to understand this new bifurcation structure. In particular, we are able to identify key characteristics that lead to the segmentation of the snaking branch and therefore provide insight into how the bifurcation structure changes with the spatial dimension. In this presentation we will focus on results pertaining to only one of the three major components of the perturbed snaking structure, as well as discuss the difficulty of extending these results to the other two.

The information dynamics of the paleoclimate
Invited talk. Saturday, 2:20 PM - 2:55 PM
Liz Bradley, University of Colorado at Boulder.

The Earth’s climate system is a nonstationary complex system with intricate spatiotemporal dynamics and complicated external forcing. One promising way to explore the dynamics of this system is to study the detailed histories that are laid down in ice cores. From the water isotope records in these cores, for instance, it is possible to reconstruct climatological factors like temperature and accumulation rates dating back to the last glacial period, and beyond. For our initial study we used the two highest-resolution records available, one from Northern Greenland (NGRIP) and one from West Antarctica (WAIS). The NGRIP core, drilled in 1999-2003, covers 128,000 years at 5cm resolution. The WAIS core, completed in the past few years, covers a shorter timespan (68,000 years), but at 0.5cm sampling.

From these data, we would like to answer questions like: Do these records contain any information about the climate system? If so, what information can we reliably extract? Are there different regimes in these data? Do extreme events like super volcanic eruptions or abrupt temperature transitions (e.g., Dansgaard-Oerstchger events), have detectable signatures? As a first pass at answering these questions, we used weighted permutation entropy (WPE) to calculate the Shannon entropy rate in a sliding window across these records. WPE is a measure of the average rate at which new information—unrelated to anything in the past—is produced by the system from which a time series is sampled. Our preliminary results suggest that analytical techniques, as well as thermodynamic, climactic, and glaciological effects, impact the information production of the climate system. For instance, WPE can detect differences in hydrogen and oxygen isotope records that are likely related to interesting geoscientific effects: kinetic fractionation in the hydrologic cycle, including evaporation of source waters, diffusion in the firn column, and solid diffusion during geothermal heating. The second-order thermodynamic differences between these isotopes are known in theory, but detecting these effects in data has been elusive until now. Additionally, studying information production over time in these records has allowed us to detect extreme events that were not visually apparent in the raw data, such as instrumentation failure and super volcanic eruptions.

Because of the physical and chemical processes that affect the ice, such as compression and deformation, the relationship between the depth in the core and the age of the material at that depth is nonlinear. Since the precise nature of those effects is unknown, it is a real challenge to deduce an age-depth model; this process involves a combination of layer counting, synchronization with tiepoints (e.g., eruptions), modeling, and interpolation. The intertwined mechanics of age, measurement resolution, accumulation variation and the art of age-depth models have created interesting challenges for us, which we will discuss in our talk.

Crystallization in a far-from-equilibrium system of sheared hard spheres
Contributed talk, Saturday, 2:55 PM - 3:15 PM
Harry L. Swinney, University of Texas at Austin.

One-half century ago a classic experiment by G.D. Scott (Nature 188, 908, 1960) showed that pouring steel balls into a rigid container filled the volume to an upper limit of 64% of the container volume, which is well below the 74% volume fraction filled by spheres in a hexagonal close packed (HCP) or face centered cubic (FCC) lattice. Many subsequent experiments have confirmed the “random closed packed” fraction of about 64%. How-
ever, the physics of the random-closed-pack limit has remained a mystery. In an experiment on a cubical box filled with 49400 weakly sheared precision glass spheres, we observe that the disordered packing compacts approximately logarithmically with time during the first 20000 shear cycles. Then a plateau corresponding to a volume fraction of 64.5% is reached, and the plateau persists for about 50000 shear cycles. Then the first growing nucleus appears, indicating a first order phase transition to a third distinct region, where crystallites of FCC and HCP symmetry emerge and coexist with the amorphous bulk. Most crystallites with ten or fewer spheres dissolve, while larger crystallites grow; all nuclei start their growth at least 10 sphere diameters distance away from any wall. By the end of our experiment, after more than two million shear cycles, nuclei with up to \( \sim 600 \) spheres are present, and 9% of all spheres are in crystallites, which have FCC or HCP symmetry with approximately equal probability. A movie illustrates the nucleation process.

**Hairy hydrodynamics**  
Invited talk, Saturday, 3:45 PM - 4:20 PM  
Peko Hosoi, MIT.

Flexible slender structures in flow are everywhere. While a great deal is known about individual flexible fibers interacting with fluids, considerably less work has been done on fiber ensembles - such as fur or hair - in flow. These hairy surfaces are abundant in nature and perform multiple functions from thermal regulation to water harvesting to sensing. Motivated by these biological systems, we consider three examples of hairy surfaces interacting with flow: (1) air entrainment in the fur of diving mammals, (2) viscous entrainment in drinking bats, and (3) symmetry breaking in hairy micro-channels. In the first example, we take inspiration from semi-aquatic mammals (such as fur seals, otters, and beavers) which have specially adapted fur that serves as an effective insulator both above and below water. Many of these animals have evolved pelts that naturally entrap air when they dive. In this study we investigate diving conditions and fur properties which amplify air entrainment. In the second example we consider viscous dipping, a feeding method utilized by many nectar drinking animals, whereby fluid is visously entrained on the surface of a tongue. This mechanism is reminiscent of Landau-Levich-Derjaguin (LLD) dip coating, and has been analyzed through this framework in previous studies. However, many viscous dippers have hairy structures on their tongues that enhance fluid uptake. Here we investigate the impact of mesoscale hairy structures on feeding efficiency. Finally, we consider a fundamental component in hydraulic systems, the flow rectifier. In particular we propose a design that allows the operator to modulate the relative resistances in the rectifier and that can be achieved using only solid state components (i.e. no moving parts).

**Characterizing impending transitions in complex systems**  
Contributed talk, Saturday, 4:20 PM - 4:40 PM  
Thomas Bury, University of Waterloo, Canada.

Co-authors: Chris T. Bauch, University of Waterloo, Canada; Madhur Anand, University of Guelph, Canada.

Tipping points - thresholds that partition drastically different system regimes - are ubiquitous in ecology, climate, finance and many other natural and artificial systems exhibiting non-linear behaviour. Within the framework of dynamical systems, tipping points correspond to bifurcations, specifically those of a sub-critical nature whereby the occupied stable state undergoes a discontinuous change. In 2009, Scheffer and colleagues proposed a set of early warning signals for tipping points, centred around the phenomenon of critical slowing down (an increasing recovery time following a disturbance), that takes place as a system approaches a bifurcation. For systems subjected to stochasticity, critical slowing down is manifested as an increased variance and autocorrelation in time-series data, which could provide warning of an impending transition. However, critical slowing down is not specific to the nature of the bifurcation and therefore does not dependably characterise system dynamics in the post-bifurcation regime. We demonstrate analytically and via simulation, how the power spectral density of time-series data leading up to a bifurcation can reveal distinct features of the post-bifurcation dynamics, which is an invaluable asset for policy makers deciding on appropriate intervention strategies. We conclude with prospects and limitations of bifurcation forecasting from noisy time-series data and highlight important avenues of research for mathematicians in this exciting, high-impact area of research.