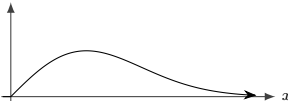# APPM 3570/STAT 3100 — Exam 2 — Spring 2024

On the front of your bluebook, write (1) **your name**, (2) **Exam 2**, (3) **APPM 3570/STAT 3100**. Correct answers with no supporting work may receive little or no credit. Books, notes and electronic devices of any kind are not allowed. Your exam should be uploaded to Gradescope in a PDF format (Recommended: **Genius Scan**, **Scannable** or **CamScanner** for iOS/Android). **Show all work, justify your answers. Do all problems.** Students are required to re-write the **honor code statement** in the box below on the **first page** of their exam submission and **sign and date it**:

1. [EXAM02] (32pts) There are 4 unrelated parts to this question. Justify your answers.

   (a) (8pts) On a stretch of highway, the number of automobile accidents occur with a Poisson distribution at an average of three accidents per week. Calculate the probability that there are at most 2 accidents occurring in any given week on this stretch of the highway.

   (b) (8pts) In a small city, the number of automobiles accidents occur with a Poisson distribution at an average of three accidents per week. What is the probability that we have to *wait* at least two weeks between any 2 accidents?

   (c) (8pts) Find the probability $P(-13 \leq X \leq 19)$ where X is a Normal random variable with parameters $\mu = -1$ and $\sigma^2 = 16$. Give your answer in terms of Φ, the cumulative distribution function of the Standard Normal rv.

   (d) (8pts) Two candies are selected at random from a jar containing three M&M's©, two Reese's Pieces™ and four Smarties®. If X and Y are, respectively, the number of M&M's and Reese's Pieces included among the two candies drawn from the jar, find the *joint probability mass function* of X and Y. (Be sure to define the pmf for all real numbers.)

2. [EXAM02] (40pts) *(Wind Turbines)* Let X denote the *vibratory stress level* (psi) on a wind turbine blade at a particular wind speed in a wind tunnel. Analysis of blade stress data collected by Department of Energy wind turbines at Rocky Flats suggests that the appropriate distribution model is the *Rayleigh distribution* with probability density function given by

$$f_X(x) = \begin{cases} \dfrac{x}{\theta^2} e^{-\frac{x^2}{2\theta^2}}, & \text{if } x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is called the *scale parameter* with $E[X] = \theta \sqrt{\dfrac{\pi}{2}}$ and $\mathrm{Var}(X) = \dfrac{4 - \pi}{2} \theta^2$.

   (a) (10pts) Find F_X, the *cumulative distribution function* of X and then *verify* that $f_X(x)$ is a legitimate probability density function. (Be sure to define the cdf for all real numbers.)

   (b) (10pts) Suppose $\theta = 10$, what is the probability that the vibratory stress level is between 10psi and 20psi?

   (c) (10pts) Suppose $g(x) = 2x^2 + 20$ find $E[g(X)]$. *(Hint: No integrals need to be calculated to answer this part.)*

   (d) (10pts) If $Y = 1 - e^{-X^2/(2 \cdot 10^2)}$, find the *probability density function* of Y. *(Hint: Note that $0 < Y < 1$.)* Be sure to define the pdf for all real numbers.

PROBLEM #3 ON THE OTHER SIDE

3. [EXAM02] (28pts) Suppose that X and Y are *jointly distributed random variables* with *joint probability density function* given by

$$f(x, y) = \begin{cases} \dfrac{2}{5}(2x + 3y), & \text{if } 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) (7pts) Set-up, but *do not solve* an integral (or integrals) to find $P(X < \frac{1}{2}, Y > \frac{1}{3})$.

(b) (7pts) Find the $f_X(x)$, *marginal probability density function* of X. (Be sure to define the pdf for all real numbers.)

(c) (7pts) Find the expectation $E[X]$.

(d) (7pts) Set-up, but *do not solve* an integral (or integrals) to find $P(XY < \frac{1}{4})$.