On the front of your bluebook, write (1) your name, (2) Exam 1, (3) APPM 3570/STAT 3100. Correct answers with no supporting work may receive little or no credit. Books, notes and electronic devices of any kind are not allowed. Your exam should be uploaded to Gradescope in a PDF format (Recommended: Genius Scan, Scannable or CamScanner for iOS/Android). Show all work, justify your answers. Do all problems. Students are required to re-write the honor code statement in the box below on the first page of their exam submission and sign and date it:

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work. Signature: Date: $\qquad$

1. (40pts) There are 4 unrelated parts to this question. Justify your answers.
(a) (10pts) How many different permutations of the letters in the word TALLAHASSEE are there?
(b) (10pts) How many of the permutations of the letters in the word TALLAHASSEE have no adjacent A's?
(c) (10pts) Assuming we have a standard 52 card deck, how many ways can we select a five-card poker hand that contains exactly two pairs (that is, two of each of two different kinds and a fifth card of a third kind)?
(d) (10pts) You pick one card from a standard 52 card deck. If $E, F$ and $G$ denote the events

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E: \text { The card is a spade, } \quad F: \text { The card is red, } \quad G: \text { The card is a picture card (i.e., a } \mathrm{J}, \mathrm{Q} \text { or } \mathrm{K}) \text {. }
$$

Find $\mathrm{P}(E \cup F \cup G)$ using the Inclusion-Exclusion Principle. Simplify your answer.
2. (32pts) Ralphie can get to work in three different ways: by automobile, bus, or bicycle. On any given day that she has to go to work there is a $50 \%$ chance that she will be late when she drives her automobile. If she takes the bus, which uses a special lane reserved for buses, there is a $20 \%$ chance that she will be late. The probability that she is late when she rides her bicycle is only $10 \%$.
(a) (8pts) Chip has been keeping records. Chip knows that Ralphie drives $30 \%$ of the time, takes the bus only $10 \%$ of the time, and takes her bicycle $60 \%$ of the time. Chip wants to know what is the probability that Ralphie is late?
(b) (8pts) Ralphie is late today, Chip wants to know what is the probability that Ralphie drove her automobile?
(c) (8pts) Chip discovers that Ralphie has decided to take the bus to work for the next five days of work. Chip wants to know what is the probability that Ralphie will be late less than two times in those five days of work? State any assumptions you are making.
(d) (8pts) Let $A$ be the event that Ralphie drives her automobile to work and let $L$ be the event that Ralphie is late, find the conditional probability $P\left(L \mid A^{c}\right)$.
3. (28pts) Suppose that a fair, six-sided die is rolled twice. Let the random variable $Y$ denote the value of the first roll minus the value of the second roll.
(a) (7pts) What are all the possible values that the random variable $Y$ can take on with positive probability?
(b) $(i)(5 \mathrm{pts})$ Find the probability mass function (pmf) of $Y$. (Your pmf should be defined for all real numbers.) (ii)(2pts) Verify that your answer is a true probability mass function.
(c) $(7 \mathrm{pts})$ Find the probability that $Y^{2}+Y=2$.
(d) ( 7 pts ) If $Y$ is even, what is the probability that the first roll is a 1 ? (The even integers are defined to be $0, \pm 2, \pm 4, \ldots$.)

