APPM 3570/STAT 3100 — Exam 1 — Fall 2024

On the front of your bluebook, write (1) **your name**, (2) **Exam 1**, (3) **APPM 3570/STAT 3100**. Correct answers with no supporting work may receive little or no credit. Books, notes and electronic devices of any kind are not allowed. Your exam should be uploaded to Gradescope in a PDF format (Recommended: **Genius Scan**, **Scannable** or **CamScanner** for iOS/Android). **Show all work, justify your answers.** **Do all problems.** Students are required to re-write the **honor code statement** in the box below on the **first page** of their exam submission and **sign and date it**:

On my honor, as a University of Colorado Boulder student, I have neither given nor received unauthorized assistance on this work. Signature:____________________________ Date:_________

1. [Exam01] (40pts) Answer all the problems below. Justify your answers.

   (a) (10pts) In how many ways can we distribute eight identical balls into four distinct urns so that the fourth urn has either two or three balls in it? (The other three urns don't all have to have balls in them.)

   (b) (10pts) (*Water management*) In the town of Niwot, each month of the year receives a different level of rain with the ranking, $A, B, C, ..., K, L$, where A is the month with most rain and L is the month with the least rain. For example, $ABCDEFGHIJKL$ is a year in which January receives the most rain, February the second most, and so on. How many distinct rainfall rankings are there?

   (c) (10pts) (Referring to the *Water management* scenario above.) The town of Niwot floods if the three heaviest months of rain (A, B, C) are adjacent to one another (e.g., BCA, ACB, etc.). Assuming each ranking is equally likely, what is the probability the town floods in a given year? Ignore effects from the previous year.

   (d) (10pts) Suppose that events V and W are independent with $P(V \cup W) = \frac{6}{10}$ and $P(V) = \frac{3}{10}$, find $P(W)$.

2. [Exam01] (32pts) Drawers A and B contain green and red pencils as follows:

   Drawer A: 6 green pencils and 2 red pencils,
   Drawer B: 2 green pencils and 10 red pencils.

   A drawer is chosen at random and then a pencil is also chosen at random from that drawer. (Let A be the event that Drawer A is selected and let B be the event that Drawer B is selected, let G be the event a green pencil is selected and let R be the event a red pencil is selected.) Do the following problems:

   (a) (8pts) Given that Drawer A is selected, what is the probability of choosing a green pencil?

   (b) (8pts) Suppose we don't know which drawer is selected, what is the probability that the pencil chosen is green?

   (c) (8pts) If the pencil chosen is green, what is the probability that it was chosen from Drawer A?

   (d) (8pts) Find $P(A|G^c)$.

PROBLEM #3 ON THE OTHER SIDE

3. [EXAM01] (28pts) Let H be the event that a coin shows *heads* and T if *tails*. A fair coin is tossed independently and repeatedly until the pair HT appears (in this order) for the first time. For example, if the event is accomplished in $k=3$ flips, then this event could happen in two ways: $E = \{HHT, THT\}$.

   (a) (7pts) **Using the set notation established above**, list all the possible ways the event that HT appears (in this order) for the first time can occur for $k=6$ flips.

   (b) (7pts) Let X be the random variable denoting *the total number of coin flips needed for this event to occur*. Clearly $X \in \{2, 3, \ldots\}$ and, for example, $X(THHT) = 4$. Find the probability $P(X = 7)$.

   (c) (7pts) Find the *probability mass function* of X, $p(k) = P(X=k)$. Be sure to define the pmf for all $k \in \mathbb{R}$. *(You do not have to verify the pmf.)*

   (d) (7pts) Find the conditional probability $P(X=7 \,|\, X \geq 3)$.