Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one page of notes. You cannot collaborate on the exam or seek outside help, nor can you use the recorded lectures, a calculator, any computational software, or material you find online.

Name:

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1. (10 points, 2 each) If the statement is always true then write **TRUE**; if it is possible for the statement to be false then write **FALSE**. No justification is necessary.

   (a) The transformation $L : \mathbb{R}^{3\times 3} \to \mathbb{R}^{3\times 3}$ defined by $L(A) = \dfrac{1}{2}(A + A^T)$ is a linear transformation.

   (b) Every $n \times n$ rank 1 matrix can be written as $uv^T$ for some $u, v \in \mathbb{R}^n$.

   (c) If $A$ has Jordan Canonical Form $J$ then $A^2$ has Jordan Canonical Form $J^2$.

   (d) If $q(x) = x^T Kx - 2x^T f + c$ and $K$ is positive semi-definite, then there are infinitely many vectors that minimize $q(x)$

   (e) Let $K = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ and $q(x) = x^T Kx$. The minimum value of $q(x)$ on the unit circle is $-2$.

2. (20 points) A linear transformation $L : \mathbb{R}^3 \to \mathbb{R}^2$ is represented in the standard basis by $A = \begin{pmatrix} 1 & -3 & 0 \\ -5 & -1 & -4 \end{pmatrix}$.

   (a) Find the matrices $S$ and $T$ such that $B = T^{-1}AS$ is in canonical form.

   (b) Let $b = \begin{pmatrix} 6 \\ -2 \\ 4 \end{pmatrix}$. Find the coordinates of $L(b)$ in the new basis.

3. (25 points) Let $A = \begin{pmatrix} 3 & -4 & 1 & -1 \\ 0 & -1 & 0 & 0 \\ 3 & -5 & 3 & -2 \\ 2 & -5 & 2 & 0 \end{pmatrix}$. Can $A$ be diagonalized? If it can, find the matrices $\Lambda$ and $S$ such that $A = S\Lambda S^{-1}$. If it cannot, find the Jordan Canonical form of $A$ and the matrix $S$ such that $A = SJS^{-1}$. You do not need to calculate $S^{-1}$ in either case.

4. (20 points) Find $e^{At}$ where $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

5. (25 points) Let $A$ be the matrix with the Singular Value Decomposition given by

$$A = \begin{pmatrix} 2/\sqrt{10} & 2/\sqrt{10} \\ -1/\sqrt{10} & 1/\sqrt{10} \\ -2/\sqrt{10} & 2/\sqrt{10} \\ 1/\sqrt{10} & 1/\sqrt{10} \end{pmatrix} \begin{pmatrix} 4\sqrt{5} & 0 \\ 0 & 2\sqrt{5} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{pmatrix}$$

(a) (3 points) What is the rank of $A$?

(b) (3 points) Is $A$ singular or nonsingular?

(c) (9 points) Find the best rank 1 approximation of $A$.

(d) (10 points) Find the pseudoinverse of $A$ and use it to calculate the least squares solution to $Ax = b$ where $b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$