Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You are allowed to use one page of notes. You cannot collaborate on the exam or seek outside help, nor can you use the recorded lectures, a calculator, any computational software, or material you find online.

Name:

---

1. (10 points, 2 each) If the statement is always true then write **TRUE**; if it is possible for the statement to be false then write **FALSE**. No justification is necessary.

   (a) A square matrix always commutes with its transpose.

   (b) If A is invertible and $c \in \mathbb{R}$ is non-zero, then $(cA)^{-1} = \frac{1}{c}A^{-1}$

   (c) If A and B are n $\times$ n symmetric matrices, then AB is also symmetric.

   (d) If A is skew-symmetric, then $\det(A) = 0$.

   (e) For matrices A and B, $AB = O$ implies either $A = O$ or $B = O$.

2. (25 points) Let A $= \begin{pmatrix} 2 & -6 & 4 \\ -1 & 3 & 1 \\ 1 & -5 & 6 \end{pmatrix}$ and $b = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$

   (a) (15 points) If A is regular find its LU factorization. If A is not regular but is non-singular, find a permuted LU factorization.

   (b) (5 points) Find the determinant of A using the factorization from (a).

   (c) (5 points) Solve the equation $Ax = b$.

3. (20 points) The following two questions are unrelated.

   (a) (10 points) Are the polynomials $\{x^2+1, x^2-1, x\}$ a basis for $\mathcal{P}^2$, the vector space of all polynomials with degree ≤ 2? Justify your answer.

   (b) (10 points) Find a basis for the span of $\left\{ \begin{pmatrix} 1 \\ 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 7 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 5 \\ 1 \end{pmatrix} \right\}$.

   Are these vectors linearly independent? Justify your answer.

4. (20 points) Consider the vector spaces C^1, consisting of all continuously differentiable scalar functions f, and $\mathcal{M}_{3\times 3}$, the set of all 3×3 matrices. Determine if the sets of functions or matrices which satisfy the following conditions are subspaces.

   (a) (5 points) $f'(x) = f(x) + x$

   (b) (5 points) $f(-x) = e^{2x} f(x)$

   (c) (5 points) 3×3 matrices of the form $\begin{pmatrix} a & 0 & b \\ 0 & ab & 0 \\ b & 0 & a \end{pmatrix}$ where $a, b \in \mathbb{R}$.

   (d) (5 points) Singular 3×3 matrices.

5. (25 points) Let $A = \begin{pmatrix} 1 & -2 & -6 & 9 \\ 1 & 2 & 2 & -1 \\ -1 & 0 & 2 & -4 \\ -1 & 2 & 6 & -9 \end{pmatrix}$

   (a) (7 points) Find a basis for the image of A

   (b) (7 points) Find a basis for the kernel of A

   (c) (6 points) Find a basis for the coimage of A

   (d) (5 points) What is the rank of A^T?