Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You may use your notes, the course textbook, and the recorded lectures for this exam, but no other resources. You are not allowed to collaborate on the exam or seek outside help, nor can you use any other notes, other books, a calculator, any computational software, or any other materials you find online. Be sure to submit your work to Gradescope by 11:59pm (Mountain Time) on Friday July 22.

Name:

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1. (32 points: 8 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and provide a **counterexample.**

   (a) Suppose that A is a square matrix with real entries, real eigenvalues, and an orthogonal eigenbasis. True or False: A is symmetric.

   (b) Suppose that Q is a real $n \times n$ orthogonal matrix and $A = QB$ where A and B are both real $n \times p$ matrices. True or false: A and B have the same singular values.

   (c) Consider $A\mathbf{x} = \mathbf{b}$. The least squares solution of $A\mathbf{x} = \mathbf{b}$ is unique.

   (d) Consider the quadratic equation given by

   $$p(\mathbf{x}) = \mathbf{x}^T \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} \mathbf{x} - 2\mathbf{x}^T \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 3.$$

   True or false: It has an absolute minimum value of 2.

2. (24 points) Consider the linear system $A\mathbf{x} = \mathbf{b}$ where

   $$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}.$$

   (a) Is $A\mathbf{x} = \mathbf{b}$ consistent?

   (b) Does $\mathbf{b}$ lie in the image of A?

   (c) Find the least squares solution of $A\mathbf{x} = \mathbf{b}$.

   (d) What is the orthogonal projection of $\mathbf{b}$ onto the image of A?

   (e) What is the closest point to $\mathbf{b}$ in the image of A?

   (f) Including multiplicities, what are the singular values of A?

3. (22 points) Consider

   $$A = \begin{bmatrix} -2 & 0 & 0 \\ -3 & -2 & -3 \\ 3 & 0 & 1 \end{bmatrix}.$$

   (a) Is A complete?

   (b) Use a factorization of A to help determine A^{112}. (Your final answer should be written as a 3×3 matrix, but the entries of the matrix do not need to simplified. Reminder: you may not use a calculator or computational software on this exam.)

(c) Find e^{tA}

4. (22 points) Consider the following data:

t	1	3	4	7
y	2	0	-3	1

(a) Find the line that best fits the data in the least-squares sense, where we assume that t is the predictor.

(b) Use your line from (a) to predict the value of y when $t = 5$. (Provide an exact answer.)