This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You may have one page of notes to use on this exam. You are not allowed to collaborate on the exam or seek outside help, nor can you use any other notes, the book, the recorded lectures, a calculator, any computational software, or material you find online.

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(1) (40 points: 8 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and either provide a justification or a counterexample.

(a) The determinant of a Householder reflection matrix is 1.

(b) A singular value of $B = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$ is 2.

(c) If a square matrix has only one eigenvalue, then that matrix is complete (diagonalizable).

(d) If a square matrix is singular, then 0 is an eigenvalue of that matrix.

(e) Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear function where $T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. Then, $T \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(2) (20 points) Consider $\mathcal{W} = \text{Span} \left\{ \begin{bmatrix} 4 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ -2 \\ 2 \end{bmatrix} \right\}$. Find the the closest point in $\mathcal{W}$ to $\begin{bmatrix} 1 \\ -4 \\ -1 \\ 2 \end{bmatrix}$. (You do not need to simplify your final answer.)

(3) (20 points) Determine a symmetric matrix with eigenvalue 2 that has corresponding eigenvector $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$ and eigenvalue -6 that has corresponding eigenvector $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

(4) (20 points) Determine if $p(x, y) = 2x^2 + 2xy + 2y^2 + 4x + 2y - 3$ has a unique absolute minimum value. If it does, determine the minimizer and the absolute minimum value.