Write your name below. This exam is worth 100 points. On each problem, you must show all your work to receive credit on that problem. You may have one page of notes to use on this exam. You are not allowed to collaborate on the exam or seek outside help, nor can you use any other notes, the book, the recorded lectures, a calculator, any computational software, or material you find online.

Name:

1. (28 points: 7 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and provide a counterexample.

   (a) If U is an upper triangular matrix and has an inverse, then its inverse is lower triangular.

   (b) If the rank of A is 2 and A is a 2×4 matrix, then the kernel of A has dimension 2.

(c) If $\det(A) = 0$, then $A\mathbf{x} = \mathbf{b}$ has no solutions.

(d) If $\mathcal{W}$ is the set of $n \times n$ matrices, A, such that $\det(A) = 0$, then $\mathcal{W}$ a subspace of $\mathbb{R}^{n\times n}$, the vector space of all $n \times n$ matrices.

2. (16 points) Consider $A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -2 & -1 \\ 1 & 2 & -4 & -3 \\ 0 & 3 & -6 & -2 \end{bmatrix}$. A is row equivalent to $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and A^T is row equivalent to $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Use this information to determine bases for the four fundamental subspaces associated with the matrix A. (Clearly indicate which basis belongs to which subspace.)

3. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 5 \\ 2 & 1 & 2 \end{bmatrix}$.

   (a) (14 points) Use Gauss-Jordan Elimination to find the inverse of **A**.

(b) (6 points) Use your answer from (a) to find the solution of

$$\begin{cases} x_1 + 2x_2 + 3x_3 = -3 \\ 3x_1 + 5x_2 + 5x_3 = 0 \\ 2x_1 + x_2 + 2x_3 = 2 \end{cases} .$$

You must use your answer from (a) to receive points. (Other methods will receive no points.)

4. (15 points) Prove that if A is nonsingular, then $A^T A$ is also nonsingular.

5. Let $A = \begin{bmatrix} 0 & 0 & -4 \\ 1 & 2 & 3 \\ 0 & 1 & 7 \end{bmatrix}$.

   (a) (15 points) Determine the permuted LU-factorization of A.

   (b) (6 points) Use the answer from (a) to find the determinant of A. (Other methods will receive no credit here.)